Total number of printed pages–7

3 (Sem–4/CBCS) PHY HC 1

2024

PHYSICS

(Honours Core)

Paper : PHY–HC–4016

(Mathematical Physics–III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) What is the smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$?

(b) What is Argand diagram ?

(c) State Taylor's theorem.

Contd.

(d) State convolution theorem of Fourier transform.

(e) Name *any two* branches of physics where tensors are applied.

(f) Find the Laplace transform of the function $f(t) = 1$.

(g) Write down the conditions for existence of Fourier transform.

2. Answer the following questions : 2×4=8

(a) Express the following complex number in polar form and plot in Argand diagram

$$2 + 2\sqrt{3}\,i$$

(b) Find Laplace transform of the function $F(t) = 3e^{3t} + 5t^4 - 4\cos 2t$

(c) Check whether the complex function $f(z) = \frac{1}{2}$ is analytic or not.

(d) Prove that $\partial_{ij}\varepsilon_{ijk} = 0$.

3. Answer ***any three*** questions of the following : 5×3=15

(a) Show that the real and imaginary parts of the function $w = log z$ satisfy the Cauchy-Riemann equations when z is not zero. Find its derivative. 3+2=5

(b) Define Fourier transform of a function $f(x)$. Find Fourier transform of $e^{-x^2/2}$. What is your inference ? 1+3+1=5

(c) Evalute $\int_C (z - z^2)\,dz$, where C is upper half of the circle $|z| = 1$. What is the value of this integral if C is the lower half of the above circle ? 3+2=5

(d) Using Laplace transform, find the solution of the initial value problem

$$y'' + 9y = 6\cos 3t\,,\ y(0) = 2,\ y'(0) = 0$$

(e) What are raising and lowering of indices of a tensor? Prove that the two operations of raising and lowering the indices are reciprocal to each other.

2+3=5

4. Answer ***any three*** of the following questions :

10×3=30

(a) (i) Obtain the Cauchy-Riemann conditions for the function $f(z) = u + iv$ to be an analytic function where u and v are the functions of x and y. Are the conditions sufficient? 5+1=6

(ii) Find the first *three* terms of the Taylor series expansion of the complex variable function

$$f(z) = \frac{1}{z^2 + 4} \text{ about } z = -i.$$ 4

(b) Evaluate the following integrals using calculus of residues : ***(any two)***

5+5=10

(i) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{\left(1 + x^2\right)^2} dx$

(ii) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$

(iii) $\displaystyle\int_0^{\infty} \frac{\sin x}{x} dx$

(c) State and prove Fourier integral theorem.

(d) (i) Applying change of scale theorem, find

$$L[\sin 3t].$$ 2

(ii) By the Laplace transform method, develop the formal solution of the differential equation which characterizes the motion of a damped harmonic oscillator. 8

(e) (i) Show that $\frac{\partial x^p}{\partial x^q} = \delta^p_q$ 1

(ii) Show that the components of kronecker delta δ^i_j do not change under coordinate transformation. 4

(iii) A covariant tensor has components xy, $2y - z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates. 5

(f) (i) Find the inverse Laplace transform

$$\frac{2s^2 - 4}{(s+1)(s-2)(s-3)}$$ 6

(ii) State and prove the first shifting property of Laplace transform. 4

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