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3 (Sem–5/CBCS) MAT HE 4/5/6

2024

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION – A

Paper : MAT–HE–5046

(Linear Programming)

Full Marks : 80

Time : Three hours

OPTION – B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION – C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION – A

Paper : MAT–HE–5046

(Linear Programming)

Full Marks : 80

1. Choose the correct answer : 1×10=10

(i) The optimal value of the objective function of the Linear Programming Problem (LPP),

Minimize $Z = 3x_1 + 2x_2$

subject to $x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

is –

(a) 5

(b) 6

(c) 7

(d) 8

(ii) If the objective function of a LPP assumes its optimal value at more than one extreme points of the convex set of its feasible solutions, then

(a) the LPP has no solution

(b) there exists at least one basic feasible solution which is not an extreme point

(c) the number of extreme points of the feasible region must exceed the number of basic feasible solutions

(d) every convex combination of these extreme points gives the optimal value of the objective function

(iii) A basic solution to a system of linear equations is called degenerate, if –

(a) none of the basic variables vanish

(b) exactly one of the basic variables vanish

(c) one or more of the basic variables vanish

(d) it is also a feasible solution

(iv) Which of the following is not correct ?

(a) The graphical approach to a LPP is most suitable when there are only two decision variables.

(b) Decision variables in a LPP may be more or less than the number of constraints.

(c) All the constraints and decision variables in a LPP must be of either "$\leq$" or "$\geq$" type.

(d) All decision variables in a LPP must be non-negative

(v) Which of the following is not associated with a LPP ?

(a) Proportionality

(b) Uncertainty

(c) Additivity

(d) Divisibility

(vi) A necessary and sufficient condition for a basic feasible solution to a minimization LPP to be optimal is that for all j –

(a) $z_j - c_j \leq 0$

(b) $z_j - c_j \geq 0$

(c) $z_j - c_j > 0$

(d) $z_j - c_j < 0$

(vii) Choose the incorrect statement :

(a) If the primal is a maximization problem, its dual will be a minimization problem.

(b) The primal and its dual do not have the same number of variables.

(c) Corresponding to every unrestricted primal variable there is an equality dual constraint.

(d) For an unbounded primal problem, its dual has a feasible solution.

(viii) The total transportation cost to the initial feasible solution to the transportation problem

	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
	20	40	30	10	

obtained by least cost method is –

(a) 100

(b) 180

(c) 310

(d) 796

(ix) The assignment problem is a special case of transportation problem in which the number of origins –

(a) equals the number of destinations

(b) is greater than the number of destinations

(c) is less than the number of destinations

(d) is less than or equal to the number of destinations

(x) In a two-person zero-sum game,

(a) if the optimal solution requires one player to use a pure strategy, then the other player must also do the same

(b) gain of one player is exactly matched by a loss to the other player so that their sum is equal to zero

(c) the game is said to be fair if both the players have equal number of strategies

(d) the player having more strategies to play is said to dominate the other player

2. Answer the following questions : 2×5=10

(a) Examine the convexity of the set

$$S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \le 4\}$$

(b) Explain the use of artificial variables in Linear Programming. Name *two* methods generally employed for the solution of LPP having artificial variables.

(c) Write the dual of the following LPP :

Maximize $Z = 4x_1 + 7x_2$

subject to $3x_1 + 5x_2 \le 6$

$x_1 + 2x_2 \le 8$

$x_1, x_2 \ge 0$

(d) Give the mathematical formulation of a transportation problem.

(e) Find the saddle point of the pay-off matrix –

		B	
	2	4	5
A	10	7	8
	4	5	6

3. Answer ***any four*** of the following : 5×4=20

(a) Solve the following LPP graphically :

Minimize $Z = 3x_1 + 5x_2$

subject to $2x_1 + 3x_2 \geq 12$

$$-x_1 + x_2 \leq 3$$

$$x_1 \leq 4$$

$$x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

(b) Reduce the feasible solution $x_1 = 2$, $x_2 = 4$, $x_3 = 1$ to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18$$

to a basic feasible solution.

(c) Use simplex method to solve the LPP :

Maximize $Z = x_1 + 9x_2 + x_3$

subject to $x_1 + 2x_2 + 3x_3 \leq 9$

$$3x_1 + 2x_2 + 2x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0 .$$

(d) Solve the dual of the following LPP :

Minimize $Z = 10x_1 + 40x_2$

subject to $x_1 + 2x_2 \geq 2$

$$-x_1 - x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

(e) Obtain an initial basic feasible solution to the following transportation problem by North-West Corner rule :

	D_1	D_2	D_3	D_4	
O_1	95	105	80	15	120
O_2	115	180	40	30	70
O_3	115	185	95	70	50
	50	40	40	110	

(f) Solve the following minimal assignment problem :

	I	*II*	*III*	*IV*
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

4. Define convex set. Show that the set of all convex combinations of a finite number of points is a convex set. 10

OR

Find all the basic feasible solutions of the system of equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

5. Solve the following LPP by two-phase method : 10

Maximize $Z = 5x_1 - 4x_2 + 3x_3$

subject to $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$

$x_1, x_2, x_3 \geq 0$

OR

Use Big-M method to solve the following LPP :

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3 \geq 0$

6. State and prove the Fundamental theorem of Duality. 10

OR

Solve the following transportation problem :

	D_1	D_2	D_3	
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

7. Solve the following assignment problem : 10

	I	*II*	*III*	*IV*
A	20	28	19	13
B	15	30	16	28
C	40	21	20	17
D	21	28	26	12

OR

Use Linear Programming method to solve the following game :

B

	1	−1	3
A	3	5	−3
	6	2	−2

OPTION – B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

1. Answer the following questions : 1×10=10

   (i) Define great circle and small circle.

   (ii) Define hour angle of a heavenly body.

   (iii) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero ?

   (iv) Name the *two* points in which the ecliptic cuts the equator on the celestial sphere.

   (v) What do you mean by circumpolar star ?

   (vi) State the third law of Kepler.

   (vii) Where does the celestial equator cut the horizon ?

   (viii) Define right ascension of a heavenly body.

   (ix) What are the altitude and hour angle of the zenith ?

   (x) State the cosine formula related to a spherical triangle.

2. Answer the following questions : 2×5=10

   (a) State Newton's law of gravitation.

   (b) ABC is an equilateral spherical triangle, show that $\sec A = 1 + \sec a$.

   (c) Give the usual three methods for locating the position of a star in space.

   (d) Prove that the altitude of the celestial pole at any place is equal to the latitude of the place of the observer.

   (e) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

3. Answer **any four** questions of the following : 5×4=20

   (a) In a spherical triangle ABC, prove that $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

   (b) In a spherical triangle ABC, if $b + c = \pi$, then prove that $\sin 2B + \sin 2C = 0$.

   (c) At a place in north latitude ϕ, two stars A and B of declinations δ and δ_1 respectively, rise at the same moment and A transists when B sets. Prove that $\tan\phi \tan\delta = 1 - 2\tan^2\phi \tan^2\delta_1$

(d) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi \sec\delta$, where the symbols have their usual meanings.

(e) Deduce Kepler's laws from the Newton's law of gravitation.

(f) Prove that the altitude of a star is the greatest when it is on the meridian of the observer.

4. Answer ***any four*** questions of the following : 10×4=40

(a) In a spherical triangle ABC, prove that

$$\frac{\sin a}{\sin A} = \sqrt{\frac{1-\cos a \cos b \cos c}{1+\cos A \cos B \cos C}}.$$

(b) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos\psi = \frac{\sqrt{ab}}{a+b-\sqrt{ab}}$$

(c) If the inferior ecliptic limits are $\pm\varepsilon$ and if the satellite revolves n times as fast as the sun, and its node regrades θ every revolution the satellite makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node then the integer next less than

$$\frac{2(n-1)\varepsilon}{n\theta+2\pi}.$$

(d) State Kepler's laws of planetary motion. If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively and e is the eccentricity of the planet's orbit, prove that $(1-e)V_1 = (1+e)V_2$.

(e) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

2. Answer the following questions : 2×4=8

(a) Give the output for

printf("\n%d%d%d \n", i,++i, i++)

(Assume i = 3)

(b) Explain printf ( ) function.

(c) What are the differences between break and exit ( ) function ?

(d) What is local variable and global variable ?

3. Answer ***any three*** from the following : 5×3=15

(a) What is 'for loop'? Write down the form of 'for loop'. Write a C program to check whether a given number is prime or not using 'for loop'. 1+1+3=5

(b) Explain with examples all the assignment operators. 5

(c) Differentiate between 'if-else' and 'nested if-else' statement. Write C program to find biggest of three numbers using if-else and nested if-else statement. (Write two programs separately) 1+2+2=5

(d) What is an array variable ? How does it differ from an ordinary variable ? How do you initialize arrays in C ? 2+1+2=5

(e) What is recursive function ? What are the uses of this function ? Write a C program to find the factorial of a given positive number using recursion. 1+2+2=5

4. (a) Write a C program to sort a set of n numbers in ascending order and explain the algorithm used. 5+5=10

OR

(b) Explain the unconditional control statements of C in detail. 10

5. (a) Explain the various types of functions supported by C. Give examples for each of the C functions. What are the rules to call a function ? What are actual and formal arguments ? 2+2+4+2=10

OR

(b) Explain the structure of C program in detail. 10

6. *(a)* Write a C program to compute $cos(x)$ upto 15 terms. 10

OR

(b) Write C programs to add and multiply two matrics of order (3×3).
