L1B

Total number of printed pages–7

3 (Sem–3/CBCS) PHY HC 1

2024

PHYSICS

(Honours Core)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

   (a) Define regular singular point at $x=0$ for a second order differential equation.

   (b) The function $P_n(1)$ is given as

      (i) -1

      (ii) zero

      (iii) 1

      (iv) $P_n(-1)$

   (Choose the correct option)

Contd.

(c) Define gamma function.

(d) Write the rank of a null matrix.

(e) What do you mean by unitary matrix?

(f) Write the orthogonal property of Hermite polynomials.

(g) State the Dirichlet condition for Fourier series.

2. Answer the following questions : 2×4=8

(a) If $\int_{-1}^{+1} P_n(x)\,dx = 2$, then find the value of n.

(b) If A and B are Hermitian matrices, show that $AB + BA$ is Hermitian and $AB - BA$ is skew-Hermitian.

(c) Using the property of gamma function evaluate the integral $\int_0^\infty x^4 e^{-x} dx$.

(d) If $f(x) = x\cos x$ is a function in the interval $-\pi < x < \pi$, then find the value of a_0 of the Fourier series.

3. Answer ***any three*** of the following questions : 5×3=15

(a) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r x^{k+r}$$, then show that the allowed values of k are zero and one only.

(b) Obtain the following orthogonality property of Legendre polynomial

$$\int_{-1}^{+1} P_n(x) P_m(x)\,dx = 0 \quad \text{for } m \neq n.$$

(c) (i) What is a special square matrix? 1

(ii) By using the Cayley-Hamilton theorem, compute the inverse of

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$ 4

(d) What is adjoint of a matrix? For the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ verify the theorem

$A.(Adj\,A) = (Adj\,A).A = |A|\,I$, where I is unit matrix. 1+4=5

(e) Find the Fourier series representing $f(x) = x, 0 < x < 2\pi$.

4. (a) (i) Find the eigenvalue and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}.$$ 6

(ii) Show that any arbitrary square matrix can be represented as a sum of symmetric and skew-symmetric matrix. 4

OR

(b) (i) Prove the Rodrigues' formula for Legendre's polynomials

$$P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^n - 1)^n.$$ 6

(ii) Show that

$H_0(x) = 1$ and $H_1(x) = 2x$.

2+2=4

5. (a) (i) Solve the following equation by the method of separation of variables

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y},$$

given $u(0, y) = 8e^{-3y}$ 5

(ii) Write the recursion formula of gamma function. Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$ 1+4=5

OR

(b) (i) A square wave function is represented as

$$f(x)=\begin{cases}0, & \text{for } -\pi < x < 0\\ L, & \text{for } 0 \le x < \pi\end{cases}$$

Draw the graphical representation of the given function. Show that the Fourier expansion of the function is given as

$$f(x)=\frac{L}{2}+\frac{2L}{\pi}\sum_{n=1}^{\infty}\frac{\sin nx}{n} \quad (\text{for } n \text{ odd})$$

1+6=7

(ii) Write the orthogonality conditions of sine and cosine functions. 3

6. (a) Prove the following recurrence relations : 3+2+5=10

(i) $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$

(ii) $H_n^1(x)=2xH_n(x)-H_{n+1}(x)$

(iii) $xP_n^1(x)-P_{n-1}^1(x)=nP_n(x)$

OR

(b) (i) Show that the matrix

$$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}\\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$$ is unitary.

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(ii) Find the power series solution of the differential equation

$\frac{d^2y}{dx^2}+w^2y=0$ in powers of x. 7