3 (Sem–6/CBCS) MAT HE 1

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6016

(Boolean Algebra and Automata Theory)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give very short answers to the following : $1\times10=10$

   (a) Define an ordered set.

   (b) Define a poset.

   (c) When are two elements of a poset called comparable ?

   (d) When does an ordered set become a total ordered set ?

(e) Write the absorption law of lattice.

(f) When are two lattices called isomorphic ?

(g) Find the minimal and maximal elements of the ordered set $(\{1, 2, 3, 4, 6, 8, 12\}, |)$, where | stands for divisibility.

(h) Define complement elements in Boolean algebra.

(i) Write true *or* false : "Every language accepted by a deterministic automation is accepted by a non-deterministic automaton."

(j) Draw a state diagram for an automaton which accepts the language expressed by $aa^*bb^*cc^*$.

2. Give answers to the following : 2×5=10

(a) Prove that every finite lattice is bounded.

(b) Draw the Hasse diagram for the lattice $(\{1, 3, 6, 12, 24\}, |)$, where | stands for divisibility.

(c) Draw a diagram for the Boolean expression $(x + y + z)(xy' + x'z)$.

(d) Define Alphabets in automata theory. Which are commonly used alphabets ?

(e) What is a string in automata theory ? Give an example.

3. Give answers to the following : ***(any four)*** 5×4=20

(a) Let L be a bounded distributive lattice. Show that the complement of L if exists, is unique.

(b) Let $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and consider the order relation '$\leq$' of divisibility on A. Let $B = P(S)$, the power set of S; where $S = \{a, b, c\}$ be the ordered set with order relation '$\subseteq$'. Show that $(A, \leq)$ and $(B, \subseteq)$ are isomorphic.

(c) Express $(x+y)(x'+z)$ and x in CNF of three variables x, y, z. 2½+2½=5

(d) When do you use the Quinn-McCluskey theorem in Boolean algebra? Write the four main steps in the Quinn-McCluskey algorithm. 1+4=5

(e) Name three types of indirect proofs. Prove that $\sqrt{2}$ is not a rational number. What is the type of proof applied in this context? 1½+2½+1=5

(f) What is Deterministic Finite automata? What are the elements of Deterministic Finite automata? 2+3=5

4. Give answers to the following: ***(any four)*** 10×4=40

(a) *(i)* Use Karnaugh maps to find a minimal form for the Boolean function $E(x, y) = x'y' + xy'$. 3

(ii) Show that the set of gates (AND, NOT) is functionally complete. 3

(iii) Construct a logic circuit corresponding to the Boolean function

$f(x, y, z) = xyz' + xy'z + x'yx$. Also simplify and draw a simpler logic circuit. 2+2=4

(b) Define a partial order relation in a set. Examine whether the following relations satisfy all axioms of a partial order relation. 2+4+4=10

(i) A relation ~ on the set of real numbers such as $x \sim y$ if and only if $x^3 - 4x \le y^3 - 4y$.

(ii) A relation ~ on the set R^2 such as $(a, b) \sim (c, d)$ if and only if $|ab| \ge |cd|$.

(c) *(i)* For any Boolean algebra B, show that

$$(a+b)(b+c)(c+a) = ab + bc + ca$$

for all elements a, b, c of B. 5

(ii) State and prove the De Morgan's laws in Boolean algebra. 5

(d) (i) Express $xy' + y(x' + z)$ in DNF in the variables present. 5

(ii) Express $(x + y' + z)(xy + x'z)$ in CNF in the variables present. 5

(e) Define a complemented lattice. Give an example of a complemented lattice. Show that two bounded lattices L and M are complemented if and only if $L \times M$ is complemented. 1+1+8=10

(f) Show that the mapping $f : B \to P(A)$ is an isomorphism where B is a Boolean Algebra, $P(A)$ is the power set of the set A of atoms and $f(x) = [a_1, a_2, ..., a_n]$ where $x = a_1 + a_2 + ... + a_n$ is the unique representation of $a \in A$ as a sum of atoms.

(g) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design a circuit as simple as you can which will allow current to pass when and only when a proposal is approved.

(h) Prove that a language $M(L)$ accepted by a pushdown automaton $M = (\sum, Q, s, I, \gamma, F)$, is a context-free language, where $\sum$ is a finite alphabet, Q is a finite set of states, s is the initial state, I is a finite of stack symbols, γ is the transition relation and F is the set of acceptance states.
