3 (Sem–6/CBCS) MAT HE 7

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6076

(Mathematical Finance)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed: 1×10=10

   (a) What do you mean by 'treasury rates'?

   (b) Write the full form of OTC.

   (c) "An interest rate in a particular situation defines the amount of money borrower promises to pay the lender."

   (Write True **or** *False)*

(d) What is arbitrage ?

(e) Fill up the blank space :

A relatively simple derivative is a ____.

(f) Write the names of main types of traders executing trades.

(g) What is implied Volatility ?

(h) What is meant by the 'Rho' of a portfolio of options ?

(i) "Suppose a 5 year zero rate with continuous compounding is quoted as 5% per annum." Write it mathematically.

(j) What is meant by the 'delta' of a stock option ?

2. Answer the following : 2×5=10

(a) Write the difference between simple interest and compound interest.

(b) Define investment asset.

(c) An investor receives Rs. 1,100 in one year in return for an investment of Rs.1,000 new. Find the percentage of return per annum with annual compounding.

(d) What are the formulae for u and d in terms of volatility ?

(e) Justify that CAPM is a pricing model.

3. Answer ***any four*** parts : 5×4=20

(a) Explain the binomial option pricing model with the help of an example.

(b) A stock price is currently Rs. 25. It is known that at the end of 3 months it will be either Rs. 25 or Rs. 23. If the risk-free interest rate is 9% with continuous compounding, what is the value of the stock 3 months hence ?

(c) Explain what is meant by —

(i) the 3-months LIBOR rate

(ii) the 3-months OIS rate which is higher. Why ?

(d) A debt of Rs. 25,000 is to be amortized over 7 years at 5% interest per annum compounded annually. What monthly payments will achieve this ?

(e) A stock index is currently 1,600. Its volatility is 16%. The risk-free rate is 3% p.a. for all maturities and the dividend yield on the index is 2.5%. Calculate values of u, d and p when 6 months time step is used.

(f) What is meant by the gamma of an option position ? What are the risks in the situation where the gamma of a position is highly negative and the delta is zero ?

4. Answer ***any four*** questions : 10×4=40

(a) Explain the terms :

(i) Shorting

(ii) Spot rates

(iii) Forward rates

(iv) Short rate

(b) A man deposits in a bank Rs. 20,000 at the end of each year, for 12 years. If the rate of interest is 12.5% p.a. compounded annually, what would be the sum standing to his credit at the end of that period ?

(c) Let $C(k,t)$ be the cost of a cell option on a specified security that has strike price k and expiration fine t.

Prove that—

(i) for fixed expiration time t, C (u,t) is a convex and non-increasing function of k.

(ii) for $s>0$, $C(u,t)-C(u+s,t) \leq se^{-kt}$

(d) Companies A and B have been offered the following rates per annum on a Rs. 5 lakhs 10-year loan.

	Fixed rate	*Floating rate*
Company A	5.0%	LIBOR +0.1%
Company B	6.4%	LIBOR +0.6%

Company A requires a floating rate loan, company B requires a fixed rate loan. Design a swap that will not a bank, acting as inter-mediary 0.1% p.a. and that will appear equally attractive to both companies.

(e) What does the Black-Scholes-Merton stock option pricing model assume about the probability distribution of the stock price in one year ? What does it assume about the probability distribution of the continuously compounded rate of return on the stock during the year ?

(f) (i) Why would one invest in an asset where beta is negative ?

(ii) Define swap and give an example of an interest rate swap.

(g) When compounded annually an interest rate is 12%. What is the rate when expressed with semi-annual compounding; quarterly compounding, monthly compounding, weekly compounding and daily compounding ?

(h) A company has a Rs. 2,00,000 portfolio with a beta of 1.2. It would like to use futures contracts on the Nifty 50 to hedge its risk. The index is currently standing at 8,400 and each contract is for delivery of Rs. 200 times the index. What is the hedge that minimizes risk ? What should the company do if it wants to reduce the beta of the portfolio to 0.6 ?
