## Total number of printed pages-8

### 3 (Sem-6/CBCS) PHY HE 5

#### 2025

#### **PHYSICS**

(Honours Elective)

Paper: PHY-HE-6056

(Classical Dynamics)

Full Marks: 80

Time: Three hours

# The figures in the margin indicate full marks for the questions.

- 1. Answer the following questions: 1×10=10
  - (a) What is called gyroradius? Give its mathematical expression.
  - (b) What do you mean by degree of freedom? Mention a system having two degrees of freedom.
  - (c) State the principle of virtual work.
  - (d) What is called phase space? Mention its dimensions.

- (e) What do you mean by cyclic or ignorable coordinates?
- (f) State the postulates of special theory of relativity.
- (g) Write down the assumptions taken to derive the Lorentz transformation equations of space-time.
- (h) Can a particle move through a medium with a speed greater than that of light in that medium? Justify your answer.
- (i) Define coefficient of viscosity. Mention its unit.
- (j) Why do bubbles of air or gas rise up through water or any other liquid?
- 2. Answer the following questions:  $2 \times 5 = 10$ 
  - (a) Does kinetic energy of a charged particle change when it enters a magnetic field? Justify your answer.
  - (b) Distinguish between Lagrangian and Hamiltonian formalisms.
  - (c) State time dilation in special theory of relativity.
  - (d) Show that  $d\tau = \frac{i}{c}ds$ , where  $d\tau$  is the element of proper time and ds is the length element in four space.

- (e) Why does Poiseuille's formula fail in the cases of tubes of wider bore and gases?
- 3. Answer **any four** from the following questions:  $5\times4=20$ 
  - (a) Show that an electron moving with uniform velocity follows a parabolic path in a transverse uniform electric field.
  - (b) Derive Lagrange's equations of motion for a conservative system using D'Alembert's principle.
  - (c) What do you mean by Minkowski's space? Four momentum  $p_{\mu}$  is given

by 
$$p_{\mu} = m_0 \frac{dx}{d\tau} \mu = m_0 v_{\mu}$$
.

Use this expression to obtain the following relation:

$$E^2 = p^2c^2 + m_0^2c^4,$$

where symbols have their usual meanings. 2+3=5

(d) What is called twin paradox in special theory of relativity? Discuss the paradox using space-time diagram.

2+3=5

- (e) Use Hamilton's canonical equations to derive the equation of motion of a simple pendulum. Find an expression for time period of oscillation of the pendulum. 4+1=5
- (f) (i) Obtain the expression for Reynold's number in terms of inertial force and force due to viscosity.
  - (ii) Write the law of dynamical similarity of flows of two different liquids through two geometrically similar tubes.
- 4. Answer the following questions: 10×4=40
  - (a) (i) Write down Lagrange's equations of motion for non-conservative system.
    - (ii) Derive Hamilton's canonical equations. 4
    - (iii) Using equation of motion of a particle in transverse direction in a central force field, show that angular momentum is conserved. And hence deduce Kepler's second law of planetary motion.

      3+2=5

- (b) (i) What do you understand by stable and unstable equilibria of a system? 2+2=4
  - (ii) Obtain Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium.
  - (iii) What do you mean by normal modes of oscillation?
- (c) (i) Write the equation of motion (in radial direction) of a particle in central force field. Prove the conservation of total energy E of the particle directly from the equation of motion.

  1+3=4
  - (ii) If  $r = e^{-\theta}$  describes the orbit of a particle under central force F(r),
    - show that  $F(r) \propto \frac{1}{r^3}$ .

(iii) State Hamilton's principle. Using this principle show that two Lagrangians L and  $L + \frac{dF(t)}{dt}$  give the same equation of motion.

1+2=3

Or

(d) (i) Two identical particles, each of mass m are attached to three identical springs, each of stiffness constant k as shown in the figure. Show that the system undergoes simple harmonic oscillations when one of the particles is slightly displaced from equilibrium position.

- (ii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four dimensional Minkowski's space.
- (e) (i) Write brief notes on space-like and time-like intervals. 2+2=4
  - (ii) Show that in four-space, the Lorentz transformation equations involve transforming from orthogonal to non-orthogonal system.

(iii) Give a geometrical interpretation of the length contraction in four-space.

Or

(f) (i) If the Lagrangian of a conservative system does not contain time explicitly, show that

 $H = \sum_{k} p_{k} \dot{q}_{k} - L$  3

(ii) A Lagrangian is given by

$$L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$$

where  $\alpha$  and  $\beta$  are constants. Deduce the Hamiltonian of the system.

- (iii) Prove that pressure and kinetic energy of a liquid are convertible from one into the other.
- (g) (i) Show that the quantity  $ds^2 = dx^2 + dy^2 + dz^2 c^2 dt^2$  is invariant under Lorentz transformation.

3

- (ii) A muon formed high up in the atmosphere travels with a speed 0.99c for a distance of 5.4km before it decays. What is the life of the muon as measured by us and as measured by the muon? 1+2=3
- (iii) Define acceleration using fourvelocity vector and hence obtain relativistic form of Newton's 2nd law of motion. 2+2=4

#### Or

- (h) (i) Obtain the velocity profile for streamline flow of a liquid through a capillary tube. Deduce the fraction of liquid which flows through the
  - section upto distance  $\frac{r}{2}$  from the axis, where r is the radius of the capillary tube.

    4+3=7
  - (ii) Three capillary tubes of lengths 8*l*, 0·2*l* and 2*l* and radii *r*, 0·2*r* and 0·5*r* respectively are connected in series. If the total pressure across the system in an experiment is *P*, deduce the pressure across the shortest (middle) capillary tube.