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**3 (Sem–4/CBCS) PHY HC 1**

**2025**

**PHYSICS**

(Honours Core)

Paper : PHY–HC–4016

***( Mathematical Physics–III )***

*Full Marks : 60*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions : 1×7=7

*(a)* State Convolution theorem on Fourier transform.

*(b)* Define simple pole singularity.

*(c)* What is residue of a complex function ?

(d) Prove $L(1)=\frac{1}{S}$, $S>0$, using definition of Laplace transform.

(e) What is the value of beta function $\beta(Z,1)$ ?

(f) Which of the following functions is analytic everywhere ?

(i) $|z|$

(ii) $Z^{-1}$

(iii) $Re\,Z$

(iv) $Sin\,z$

(g) Evaluate $\delta_q^p A_s^{qr}$.



2. Answer the following questions : 2×4=8

(a) Find the Inverse Laplace transform of $\frac{S}{S^2+1}$.

(b) Prove $\int_{-\infty}^{\infty} f(t)\delta'(t-a)dt=-f'(a)$.

(c) Obtain the Fourier transform of the function $f(x)=\begin{cases} x, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$.

(d) Write about the geometrical representation of sum of complex numbers.

3. Answer ***any three*** questions of the following : 5×3=15

(a) Using Cauchy's residue theorem, evaluate $\int_{-\infty}^{\infty} f(x)dx$.

(b) Show that the sum function of the integral formula is $\frac{1}{2}[f(x+0)+f(x-0)]$ corresponding to the function $f(x)$ in the interval $0<x<1$.

(c) Find the Laplace transformation of the function $F(t)=t^n$, $n=0,1,2,3,\ldots\ldots$.

(d) Define symmetric and antisymmetric tensors with examples.

(e) Show that in Cartesian co-ordinate system the contravariant and covariant components of a vector are identical.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Express the complex number $(1+2i)/(1-3i)$ in $r(\cos\theta+i\sin\theta)$ form. 4



(ii) Using the method of residues, show that $\int_0^{\infty}\frac{dx}{x^4+1}=\frac{\pi\sqrt{2}}{4}$. 6

(b) (i) Find the value of the integral $\int_0^{1+i}(x-y-ix^2)dz$ along real axis from $z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$. 5

(ii) Evaluate $\int_0^{2\pi}\frac{d\theta}{5-4\sin\theta}$ using calculus of residues. 5

(c) (i) What is Levi-Civita tensor ? Prove $\vec{A}\cdot\vec{B}=\sum_{i=1}^{3}\sum_{j=1}^{3}A_i B_i \delta_{ij}$. 2+3=5

(ii) Prove Cauchy-Riemann conditions for analytical functions. 5

(d) Explain co-variant and contravariant tensors.

(e) (i) Obtain Fourier's series for the expansion of $f(x) = x \sin x$ in the interval $-\pi < x < \pi$. Hence deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \ldots\ldots$ .

6+2=8

(ii) Find Fourier series for $f(x)$ in the interval $(-\pi, \pi)$ where,

$f(x) = \pi + x$, when $-\pi < x < 0$,

$f(x) = \pi - x$, when $0 < x < \pi$ . 2

(f) (i) Show that Kronecker delta is a mixed tensor of rank 2. 4



(ii) Show that the function $f(z) = \sqrt{(|xy|)}$ is not regular at the origin although the Cauchy-Riemann equations are satisfied at that point. 6

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