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3 (Sem–3/CBCS) MAT HC 3

2025

MATHEMATICS

(Honours)

Paper : MAT–HC–3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (i) What is the polar equation of a circle when the pole is at the centre ?

   (ii) Find the point on the conic $\frac{8}{r} = 3 - \sqrt{2}\cos\theta$ whose radius vector is 4.

(iii) Find the equation of the line $\frac{x}{a}+\frac{y}{b}=2$, when the origin is transferred to the point (a, b).

(iv) What is meant by diametral plane of a conicoid ?

(v) If $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is a generator of the cone represented by the homogeneous equation $f(x, y, z)$, then what is the value of $f(l, m, n)$?

(vi) Write down the conditions under which the general equation of second degree

$$ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$$

represents a sphere.

(vii) If the axes are rectangular, find the direction cosines of the normal to the plane $x+2y-2z=9$.

(viii) Under what condition

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

may represents a pair of parallel straight lines ?

(ix) Define skew lines.

(x) What is the nature of the conic represented by

$$4x^2-4xy+y^2-12x+6y+9=0 \text{ ?}$$

2. Answer the following questions : 2×5=10

(a) Find where the line $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{4}$ meets the plane $x+y+z=3$.

(b) Find the centre and foci of the hyperbola $x^2-y^2=a^2$.

(c) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2=4ax$, then prove that $t_1t_2=-1$.

4. Answer the following questions : ***(any four)***

$10\times4=40$

(a) A variable plane is parallel to the given plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$

(b) Find the equation to the cylinder generated by the lines drawn through the points of the circle $x+y+z=1, x^2+y^2+z^2=4$ which are parallel to the line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$.

(c) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0$, $z+x=0$, $x+y=0$, $x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x=y=z=-a$.

(d) Find the condition that the plane $lx+my+nz=p$ may touch the conicoid $ax^2+by^2+cz^2=1$. Verify that the plane $2x-2y+8z=9$ touches the ellipsoid $x^2+2y^2+3z^2=9$.

(e) Show that the ortho-centre of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is given by $\frac{x}{l}=\frac{y}{m}=\frac{a+b}{am^2-2hlm+bl^2}$.

(f) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(g) Show that the equation

$$9x^2-24xy+16y^2-18x-101y+19=0$$

represents a parabola and it can be reduced to the standard form $Y^2=3X$. Find the coordinates of the vertex and the focus.

(h) Find the point of intersection of the lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$