3 (Sem–3/CBCS) PHY HC 1

2025

PHYSICS

(Honours)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions : (each
   question carries **one** mark) $1×7=7$

   (a) If, $\mu^{-1}M\mu = M'$, then show that
   $Tr\, M = Tr\, M'$.

   (b) Square matrix = Symmetric matrix + ?

(c) $\beta\left(\dfrac{1}{2}, \dfrac{1}{2}\right) = ?$

(d) $\displaystyle\int_0^\infty e^{-x} x^{2n-1}\, dx = ?$

(e) Express the one-dimensional heat flow equation.

(f) $L_1(x) - L_0(x) = ?$

(g) Show that $P_n(-x) = (-1)^n P_n(x)$.

2. Answer the following questions : (each question carries **2** marks)   2×4=8

(a) Show that the matrix

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is Hermitian as well as unitary.

(b) Show that $\beta(l, m) = \beta(m, l)$.

(c) Can we express the one-dimensional Schrödinger's equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial t^2} + V\psi(x, t) = i\hbar \frac{\partial \psi}{\partial t}(x, t)$$

in terms of space dependent and time independent equations if V is a function of both x and t? Explain.

(d) Show that $x = 0$ is a regular singular point for the following differential equation :

$$2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + \left(x^2 - 4\right)y = 0$$

3. Answer **any three** questions from the following : (each question carries **5** marks)   5×3=15

(a) Show that

$$L_n'(x) - n\, L_{n-1}'(x) + n\, L_{n-1}(x) = 0.$$

(b) Find the Fourier's series representing $f(x) = x$, $0 < x < 2\pi$, and sketch its graph from $x = -4\pi$ to $x = +4\pi$.

$$3+2=5$$

(c) How many real numbers are required to express a general complex matrix of dimension 2×2? Show that a 2×2 Hermitian matrix of dimension 2×2 carries four real numbers. Also, show that a skew-Hermitian matrix of dimension 2×2 carries only the real numbers.

$$1+2+2=5$$

(d) If $\begin{pmatrix} x \\ y \end{pmatrix}$ transforms to $\begin{pmatrix} x' \\ y' \end{pmatrix}$ in the way —

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \text{ then }$$

show that $x'^2 + y'^2 = x^2 + y^2$.

Verify that the transformation matrix is orthogonal.

$$2+3=5$$

(e) By the separation of variable method, solve the t-dependent part of the following equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

4. If $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, obtain A^{-1}.

From the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix},$$

obtain, a, b, c, d. $4+6=10$

Or

Obtain the eigenvalues and eigenvectors of the matrix

$$M = \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix}$$

and hence diagonalize the same. $4+6=10$

5. If, $y = \sum_{k=0}^{\infty} a_k x^{m+k}$ happens to be the power series solution of the equation,

$2x(1-x)\dfrac{d^2y}{dx^2} + (1-x)\dfrac{dy}{dx} + 3y = 0$, then show

that $a_{k+1} = \dfrac{-2m-2k+3}{2m+2k+1}$. 10

Or

Show the following : 4+3+3=10

(1) $(n+1) P_{n+1} = (2n+1) x P_n - n P_{n-1}$

(2) $n P_n = x P_n' - P_{n-1}'$

(3) $P_{n+1}' - P_{n-1}' = (2n+1) P_n$

6. Solve the equation

$\dfrac{\partial^2 \psi}{\partial x \partial t} = e^{-t} \cos x$

given that, $\psi(t=0) = 0$ and $\dfrac{\partial \psi}{\partial t}\Big|_{x=0} = 0$.

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Or

Consider a vibrating string of length l fixed at both ends, given that

$y(0, t) = 0, \quad y(l, t) = 0$

$y(x, 0) = f(x), \quad \dfrac{\partial y}{\partial t}(x, 0) = 0; \quad 0 < x < l$

Solve completely the equation of vibrating string

$\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$.

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