Total number of printed pages–7

3 (Sem–5 /CBCS) PHY HC 1

2025

PHYSICS

(Honours Core)

Paper : PHY–HC–5016

(Quantum Mechanics and Application)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

   (a) The splitting of a spectral line in the presence of external electric field is termed as

   (i) Stark effect

   (ii) Normal Zeeman effect

   (iii) Paschen-Back effect

   (iv) Anomalous Zeeman effect

(b) What is the total number of energy level (or degeneracy) for the nth state of hydrogen atom ?

(c) What is Landé g-factor ?

(d) How does the number of superimposed waves, forming a wave packet, affect the localization of the particle ?

(e) Show that $\left[x, \frac{\delta^2}{\delta x^2}\right] = -2\frac{\delta}{\delta x}$

(f) Determine whether or not $\psi(x) = e^x$ is an acceptable wave function.

(g) Write down the quantum mechanical form of total energy operator of a particle moving in x-direction.

2. Answer the following questions : 2×4=8

(a) A particle of mass m and moving in a potential $v(x)$ has the wave function

$$\psi(x,t) = A\exp\left(-ikt - \frac{km}{\hbar}x^2\right)$$

where both A and k are constants. Determine the explicit form of the potential.

(b) Show that $\hat{p}_x = -i\hbar\frac{d}{d\phi}$ is an Hermitian operator.

(c) A particle with total energy E is influenced by a potential energy $V(x)$. Show that the one-dimensional Schrödinger equation can be written in the form

$$\left[\frac{d^2}{dx^2} + k^2 - U(x)\right]\psi(x) = 0$$

where,

$$k^2 = \frac{2mE}{\hbar^2} \quad \text{and} \quad U(x) = \frac{2mV(x)}{\hbar^2}$$

(d) What is the physical significance of the wave function $\psi(x, t)$?

3. Answer ***any three*** of the following : 5×3=15

(a) State Pauli's exclusion principle. An atomic state is denoted by ${}^4D_{5/2}$. Give the values of L, S and J. What should be the minimum number of electrons involved for the state ? 2+3=5

(b) Write down the radial wave function for 1s state of hydrogen atom. Also, compare the probabilities of a 1s electron in the hydrogen atom being at a distance a_0 from the nucleus than at a distance $a_0/2$. 1+4=5

(c) What are momentum space wave functions? Show that these wave functions can be obtained as Fourier transform of position space wave functions. 1+4=5

(d) Show that if $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are two independent solutions of the Schrödinger equation, then

$$\psi(\vec{r}) = a_1\psi_1(\vec{r}) + a_2\psi_2(\vec{r})$$

is also a solution of the Schrödinger equation. What does it imply? 4+1=5

(e) What do you mean by expectation value of a dynamical variable? Find the expectation value $<p>$ and $<p^2>$ for the wave function

$$\psi(x) = \sqrt{2/L}\ \sin\left(\frac{\pi x}{L}\right) \quad \text{for } 0<|x|<L$$

$$\text{for} \quad |x|>L$$

1+(2+2)=5

4\. (i) Describe and explain L-S coupling. Under what condition does it hold?

(ii) Under what condition L-S coupling breaks down and what kind of new coupling takes place?

(iii) Describe J-J coupling. Illustrate L-S and J-J coupling with the help of vector diagram. 3+3+4=10

Or

With a suitable diagram, illustrate the Stern-Gerlach experiment. What is the significance of inhomogeneous magnetic field used in Stern-Gerlach experiment? Explain mathematically.

In a Stern-Gerlach type experiment, the magnetic field varies with distance in z-direction according to $dB_z/dz = 1{\cdot}4\ T/mm$. Silver atoms travel a distance $x = 3{\cdot}5\ cm$ through the magnet. The speed of atoms emerging from oven is $v = 750\ m/sec$. Find the separation of the two beams as they leave the magnet. Mass of silver atom $= 1{\cdot}8 \times 10^{-25}\ kg$ and its magnetic moment is 1 *Bohr* magneton.

3+3+4=10

5. *(i)* Write down Schrödinger equation for a linear harmonic oscillator. What are the eigenvalues and eigenfunctions of the Hamiltonian of a linear harmonic oscillator ? Explain the significance of zero-point energy of the oscillator.

1+2+2=5

(ii) Find the expectation value of energy when the state of harmonic oscillator is described by the following wave function :

$$\psi(x,t) = \frac{1}{\sqrt{2}}[\psi_0(x,t) + \psi_1(x,t)]$$

where $\psi_0(x,t)$ and $\psi_1(x,t)$ are wave functions for the ground state and the first excited state respectively. 5

Or

Write down Schrödinger wave equation for hydrogen atom in spherical polar coordinates. Separate the equation into radial and two angular parts. Also, from the radial part of the Schrödinger equation, find the eigenvalues of energy E for the ground state of hydrogen atom.

1+2+7=10

6. What is the need for normalization of a wave function ? Calculate the normalization constant of a wave function (at $t=0$) given by

$$\psi(x) = a e^{\left(-\alpha^2 x^2/2\right)} . e^{ikx}$$

known as the Gaussian wave packet.

Determine *(a)* the probability density, and *(b)* the probability current density of the wave function. 2+3+5=10

OR

A finite square potential well of depth V_0 is defined as

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ -V_0 & \text{for} \quad 0 \le x \le L \\ 0 & \text{for} \quad x > L \end{cases}$$

Set up Schrödinger equation for the potential well. Also solve using appropriate boundary conditions and determine the energy eigenvalues. 2+8=10
