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3 (Sem-5/CBCS) MAT HE 4/5/6

2025

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION-A

Paper : MAT-HE-5046

(Linear Programming)

Full Marks : 80

Time : Three hours

OPTION-B

Paper : MAT-HE-5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION-C

Paper : MAT-HE-5066

(Programming in C)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

OPTION-A

Paper : MAT-HE-5046

(Linear Programming)

Full Marks : 80

Time : Three hours

1. Answer **any ten** questions from the following: (Choose the correct answer)

1×10=10

- (i) Define finite game in "Game Theory".
- (ii) In a transportation table for cell evaluation we use the formula

- (a) $c_{rs} = u_r + v_s, d_{ij} = (u_i + v_j) - c_{ij}$
- (b) $c_{rs} = u_r + v_s, d_{ij} = c_{ij} - (u_i + v_j)$
- (c) $c_{rs} = u_r + v_s, d_{ij} = u_i + v_j$
- (d) None of the above

(iii) For optimality test in a transportation problem, number of allocation in independent position must be

- (a) $m + n$
- (b) $m + n + 1$
- (c) $m + n - 1$
- (d) None of the above

(iv) In a transportation problem if we apply North-West Corner method, we always get

- (a) non-degenerated BFS
 - (b) degenerated BFS
 - (c) optimal solution
 - (d) None of the above
- (v) In an assignment problem

$$\min Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij},$$

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \text{ means that}$$

- (a) only one job is done by i -th person
 $i = 1, 2, \dots, n$
- (b) i -th person is assigned to j -th job
- (c) only one person should be assigned to the j -th job,
 $j = 1, 2, \dots, n$
- (d) None of the above

(vi) If any of the constraint in the primal is perfect equality, the corresponding dual variable is

- (a) perfect equality
- (b) unrestricted in sign
- (c) strictly inequality
- (d) None of the above

(vii) The necessary and sufficient condition for any LPP and its dual to have optimal solution is that

- (a) both have basic solution
- (b) both have unbounded solution
- (c) both have feasible solution
- (d) None of the above

(viii) If we consider dual of an LPP, then in the dual the requirement vector of the primal problem becomes

- (a) objective function
- (b) price vector
- (c) variable
- (d) None of the above

(ix) The incoming vector in a simplex table will be taken as α_k if

- (a) $\Delta_k = \max \Delta_j$
- (b) $\Delta_j = \max \Delta_k$
- (c) entries of α_k are all negative
- (d) None of the above

(x) Standard form of LPP is

- (a) $\text{Min } Z = cx \text{ s.t. } Ax \geq b, x \geq 0$
- (b) $\text{Max } Z = cx \text{ s.t. } Ax \leq b, x \geq 0$
- (c) $\text{Max } Z = cx \text{ s.t. } Ax \geq b, x \geq 0$
- (d) None of the above

(xi) If for any basic feasible solution of an LPP, there is some column α_j in A but not in B for $c_j - z_j > 0$ and

$y_{ij} \leq 0$ ($i = 1, 2, \dots, m$), then the problem has an unbounded solution if the objective function is to be

- (a) maximized
- (b) minimized
- (c) either maximized or minimized
- (d) None of the above

(xii) If the LPP $\max Z = cx$ such that $Ax = b, x \geq 0$ has a feasible solution then at least one of the BFS will be

- (a) maximal
- (b) minimal
- (c) optimal
- (d) None of the above

(xiii) A basic feasible solution of an LPP is said to be non-degenerate BFS if

- (a) none of the basic variable zero
- (b) at least one of the basic variable zero
- (c) exactly one of the basic variable zero
- (d) None of the above

(xiv) Consider a system $Ax = b$ of m equations in n unknowns, $n > m$. Then maximum number of basic solution is

- (a) ${}^m C_n$
- (b) ${}^n C_{m-1}$
- (c) ${}^n C_{n-m}$
- (d) None of the above

(xv) In a linear programming problem

$$\max Z = cx$$

subject to $Ax \geq b, x \geq 0, c$ is called

- (a) coefficient vector
- (b) column vector
- (c) price vector
- (d) None of the above

(xvi) The extreme points of the convex set of feasible solutions are

- (a) finite in number
- (b) infinite in number
- (c) either finite or infinite
- (d) None of the above

(xvii) If X is the set of eight vertices of a cube, then the convex hull $C(X)$ is the

- (a) surface of the cube
- (b) vertices of the cubic
- (c) whole cube
- (d) None of the above

(xviii) A function $f(x)$ is said to be strictly convex at x if for two other distinct points x_1 and x_2

$$(a) \quad f[\lambda x_1 + (1-\lambda)x_2] < \lambda f(x_1) + (1-\lambda)f(x_2),$$

where $0 \leq \lambda \leq 1$

$$(b) \quad f[\lambda x_1 + (1-\lambda)x_2] < \lambda f(x_1) + (1-\lambda)f(x_2),$$

where $0 < \lambda < 1$

$$(c) \quad f[\lambda x_1 + (1-\lambda)x_2] \leq \lambda f(x_1) + (1-\lambda)f(x_2),$$

where $0 \leq \lambda \leq 1$

(d) None of the above

2. Answer **any five** from the following:

$$2 \times 5 = 10$$

(a) What is cost matrix of an assignment problem?

(b) Write the 'Test of optimality' for primal dual method.

(c) How can we find entering vector in a simplex table?

(d) Write "complementary slackness theorem" of a dual problem.

(e) What is *two-phase* method to solve an LPP? Mention the phases.

(f) Define unbounded solution of an LPP. How can we determine that the solution of an LPP is unbounded?

(g) Define artificial variable. Give an example.

(h) Examine convexity of the set

$$S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\}$$

(i) Is $x_1 = 1, x_2 = \frac{1}{2}, x_3 = x_4 = x_5 = 0$

a basic solution to the following system?

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

(j) Show that the FS, $x_1 = 1, x_2 = 0, x_3 = 1$ and $z = 6$ to the system of equations

$$x_1 + x_2 + x_3 = 2$$

$$x_1 - x_2 + x_3 = 2, \quad x_j \geq 0 \text{ which minimize}$$

$$z = 2x_1 + 3x_2 + 4x_3 \text{ is not basic.}$$

3. Answer **any four** from the following :
 $5 \times 4 = 20$

(a) If in an assignment problem, a constant is added or subtracted to every element of row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimize the total cost for the other matrix.

(b) Find initial BFS of the following LPP :

$$\text{Max } Z = 2x_1 + 3x_2$$

$$\text{s.t. } -x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9$$

$$x_1, x_2 \text{ unrestricted}$$

(c) Mark the feasible region represented by the constraint conditions

$$x_1 + x_2 \leq 1, 3x_1 + x_2 \geq 3, x_1 \geq 0, x_2 \geq 0$$

(d) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(e) Solve the following transportation problem by North-West Corner method :

	S_1	S_2	S_3	S_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	40	100

(f) Give the dual of the following LPP :

$$\text{Max } Z = 2x_1 + 3x_2 + x_3$$

$$\text{s.t. } 4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

(g) If the objective function of an LPP assume its optimal value at more than one extreme point, then prove that every convex combination of these extreme points gives the optimal value of the objective function.

(h) Prove that the set of all feasible solutions of an LPP is a convex set. [Assume that the set is non-empty]

4. Answer **any four** from the following :

$$10 \times 4 = 40$$

(a) A company has four territories open and four salesman available for assignment. The territories are not equally rich in their sales potential; it is estimated that a typical salesman operating in each territory would bring in the following annual sales :

Territories	I	II	III	IV
Annual sales (Rs.)	60,000	50,000	40,000	30,000

The four salesman are also considered to differ in ability; it is estimated that, working under the same conditions, their yearly sales would be proportionally as follows :

Salesman	A	B	C	D
Proportion	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory, the next best salesman to the second richest and so on. Verify this answer by the assignment technique.

(b) If an LPP

$$\text{Max } Z = cx \text{ s.t. } Ax = b, x \geq 0$$

where A is $m \times (m+n)$ matrix of coefficients given by $A = (\alpha_1, \alpha_2, \dots, \alpha_{m+n})$, has at least one feasible solution, then prove that it has at least one basic feasible solution.

(c) Find an optimal solution of the following LPP without using simplex method :

$$\text{Max } Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

$$\text{s.t. } 2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_i \geq 0, i = 1, 2, 3, 4$$

(d) The pay-off matrix for A in a two-persons zero-sum game is given below. Determine the value of the game and the optimum strategies for both players.

		B		
		I	II	III
A	I	-1	2	1
	II	1	-2	2
	III	3	4	-3

(e) State and prove Fundamental Duality theorem.

(f) Solve the following transportation problem:

Plant	Market				Available
	A	B	C	D	
X	10	22	10	20	8
Y	15	20	12	8	13
Z	20	12	10	15	11
Required	5	11	8	8	32

(g) Use dual to solve the LPP

$$\text{Min } Z = 2x_1 + x_2$$

$$\text{s.t. } 3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

(h) Using simplex algorithm solve the problem—

$$\text{Max } Z = 2x_1 + 5x_2 + 7x_3$$

$$\text{s.t. } 3x_1 + 2x_2 + 4x_3 \leq 100$$

$$x_1 + 4x_2 + 2x_3 \leq 100$$

$$x_1 + x_2 + 3x_3 \leq 100$$

$$x_1, x_2, x_3 \geq 0$$

(i) State and prove fundamental theorem of LPP.

(j) A soft drink plant has two bottling machines A and B. It produces and sells 8 ounce and 16 ounce bottles. The following data is available.

Machine	8 ounce	16 ounce
A	100/minute	40/minute
B	60/minute	75/minute

The machines can run 8 hours per day, 5 days per week. Weekly production of drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 eight ounce bottles and 7,000 sixteen ounce bottles per week, profit on these bottles are 15 paise and 25 paise per bottle respectively. The planner wishes to minimize his profit subject to all the production and marketing restrictions. Formulate it as an LPP and solve graphically.

OPTION-B

Paper : MAT-HE-5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

1. Answer **any ten** of the following questions :
1×10=10

- (i) State the cosine formula related to a spherical triangle.
- (ii) What are the altitude and hour angle of the zenith ?
- (iii) Define right ascension of a heavenly body.
- (iv) Where does the celestial equator cut the horizon ?
- (v) Explain, what is meant by rising and setting of stars.
- (vi) What is the duration of a day and night at equinoxes ?
- (vii) Define polar triangle.

- (viii) State the third law of Kepler.
- (ix) What do you mean by circumpolar star ?
- (x) What is parallactic ellipse ?
- (xi) What is the declination of the pole of the ecliptic ?
- (xii) Name the *two* points in which the elliptic cuts the equator on the celestial sphere.
- (xiii) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero ?
- (xiv) Define hour angle of a heavenly body.
- (xv) What are the relations between the elements of a spherical triangle and that of its polar triangle ?
- (xvi) How many great circles can be drawn through two given points, when the points are the extremities of a diameter ?
- (xvii) Define great circle and small circle.
- (xviii) Define spherical triangle.

2. Answer **any five** of the following questions :
2×5=10

(a) Show that right ascension α and declination δ of the sun is always connected by the equation

$\tan \delta = \tan \varepsilon \sin \alpha$, ε being obliquity of the ecliptic.

(b) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(c) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(d) Discuss the effect of refraction on sunrise.

(e) Prove that the altitude of the celestial pole at any place is equal to the latitude of the place of the observer.

(f) Give the usual three methods for locating the position of a star in space.

(g) ABC is an equilateral spherical triangle, show that $\sec A = 1 + \sec a$.

(h) State Newton's law of gravitation.

(i) Prove that the sides and angles of a polar triangle are respectively the supplements of the angles and sides of the primitive triangle.

(j) Draw a neat diagram of the celestial sphere showing the horizontal coordinates of a heavenly body.

3. Answer **any four** questions of the following :
5×4=20

(a) Prove that the altitude of a star is the greatest when it is on the meridian of the observer.

(b) If v_1, v_2 be the velocities of two planets in their orbits, and T_1, T_2 , be the respective distances from the sun, prove that $v_2 : v_1 = \sqrt{T_1} : \sqrt{T_2}$.

(c) Write short notes on: (i) Zodiac (ii) Morning star and Evening star.

(d) Deduce Kepler's laws from the Newton's law of gravitation.

(e) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos \psi = \sin \phi \sec \delta$, where the symbols have their usual meanings.

(f) At a place in north latitude ϕ , two stars A and B of declinations δ and δ_1 respectively, rise at the same moment and A transits when B sets. Prove that $\tan \phi \tan \delta = 1 - 2 \tan^2 \phi \tan^2 \delta_1$.

(g) In a spherical triangle ABC , if $b + c = \pi$, then prove that $\sin 2B + \sin 2C = 0$.

(h) In a spherical triangle ABC , prove that $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

4. Answer **any four** questions of the following:
10×4=40

(a) Prove that, if the fourth and higher powers of e are neglected,

$$E = M + \frac{e \sin M}{1 - e \cos M} - \frac{1}{2} \left(\frac{e \sin M}{1 - e \cos M} \right)^3$$

is a solution of Kepler's equation in the form.

(b) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

(c) A star of declination δ is seen on the prime vertical. Show that its declination is increased by $\frac{2K(\sin^2 \phi - \sin^2 \delta)}{\sin 2\delta}$ due to refraction, ϕ being the latitude of the place.

(d) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k \tan \xi$, ξ being the apparent zenith distance of a heavenly body. Mention one limitation of this formula.

(e) State Kepler's laws of planetary motion.

If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively and e is the eccentricity of the planets' orbit, prove that

$$(1 - e)V_1 = (1 + e)V_2.$$

(f) Explain the effects of refraction on right ascension and declination.

(g) What is Cassini's hypothesis? Under this hypothesis, show that the amount of refraction R can be found from

$$\tan \phi = \frac{\sin R}{\mu - \cos R}, \text{ where } \mu \text{ is the}$$

refractive index of the atmosphere with respect to vacuum and ϕ is the angle of refraction at certain point on the upper surface of the atmosphere.

(h) If the inferior ecliptic limits are $\pm \varepsilon$ and if the satellite revolves n times as fast as the sun, and its node regresses θ every revolution the satellite makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node than the integer

$$\text{next less than } \frac{2(n-1)\varepsilon}{n\theta + 2\pi}.$$

(i) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos \psi = \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

(j) In a spherical triangle ABC , prove that

$$\frac{\sin a}{\sin A} = \sqrt{\frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}}$$

OPTION-C

Paper : MAT-HE-5066

(**Programming in C**)

Full Marks : 60

Time : Three hours

1. Answer **any seven** questions : $1 \times 7 = 7$

(a) Define a two-dimensional array.

(b) Write the utility of `getch ()` function.

(c) For $x = 5$, $y = 2$, write the output of $x \% y$.

(d) Write **any two** built-in functions used in C-language.

(e) State whether True **or** False :
C-language is case-sensitive.

(f) Convert the following mathematical expression into a C expression :

$$z = \frac{5x+6}{3x^2+2} - \frac{\sin x^2}{\sqrt{x}}$$

(g) Write **two** reserved words used in C language.

(h) Which of the following can be used as a variable : $x1$, x_1 , $x\%1$?

(i) Name the header file that is used to compile the function '`sqrt (x)`'.

(j) What is a global variable ?

(k) Write **one** arithmetic and **one** logical operator in C.

(l) Write the output of a :

```
int a;
```

```
a=5/2;
```

2. Answer **any four** questions : $2 \times 4 = 8$

(a) Write the output of the following :

```
c = 0
```

```
for (i = 1; i ≤ 5; i++)
```

```
    c = c + i;
```

(b) Write the syntax of 'nested if' statement in C language.

(c) Write the general syntax of `scanf()` function to read the integer variable *a*.

(d) `int a, b, temp;`

```
a = 5;
```

```
b = 3;
```

```
temp = a;
```

```
    a = b;
```

```
    b = temp;
```

Write the output of 'a' and 'b'.

(e) Write a C program that will input a character and give output, the same.

(f) Name **any four** functions available in 'stdio.h'.

(g) Write **four** different C statements each adding 1 to integer variable *x*.

(h) What is the difference between C character and C string?

3. Answer **any three** parts : $5 \times 3 = 15$

(a) Write a C program to add 1 to each element of a 3×3 matrix.

(b) Write a C program to print the $n \times n$ zero matrix.

(c) Write a C program to find the sum of squares of all integers between 1 and *n*.

(d) Write a C program to generate the Fibonacci series up to *n* terms.

(e) Write a C program to print integers from 1 to *n* omitting those integers which are divisible by 7.

(f) Give a general syntax of 'switch' statement in C.

Write the outputs of a and b of the following:

(i) $a=5;$ (ii) $a=5;$ (iii) $a=5;$
 $b=7;$ $b=7;$ $b=7;$
 $if(a > b)$ $if(a > b)$ $if(a > b || a < b)$
{ $a=a+1;$ $a=a+1;$ $a=a+1;$
 $b=b+1$ }; $b=b+1$

(g) Write a C program to find the average of best three marks from the given four test marks.

(h) Write a C program to calculate the commission for a sales representative as per the sales amount given below:
if sales \leq 500, commission is 5% of sales

if sales $>$ 500 but \leq 2000, commission is Rs. 35 plus 10% above Rs. 500 of sales

if sales $>$ 2000 but \leq 5000, commission is Rs. 185 plus 12% above Rs. 2000 of sales

if sales $>$ 5000, commission is 12.5% of sales

4. Answer **any three** parts: 10×3=30

(a) What are the uses of 'continue' and 'goto' statements in a C program? Explain each with a suitable C program segment. 5+5=10

(b) What is meant by recursive function? What is its use? Demonstrate the use of recursive function by a suitable C program. 2+2+6=10

(c) Write the syntax of 'nested for' loop and show with a suitable C program. What are the differences between 'break' statement and 'exit()' function? Write a C program using 'break' statement, and write the outputs. Also write the outputs of the same program if the 'break' statement is replaced by 'exit()' function. 1+4+2+3=10

(d) Write a C program to find the LCM of two numbers a and b , where b is the sum of the digits of a . Use two functions—one is to find LCM and the other is to find the sum of the digits.
($gcd.lcm=a.b$)

- (e) Write a C program to compute the value of e^x using the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

For this build two functions—one to find the factorial and the other to compute x^n , for a given n .

- (f) Write a C program for each of the following:

$$5+5=10$$

(i) To find the mean and standard deviation of any n values.

(ii) To add two matrices of order $m \times n$.

- (g) Develop a C program to compute the value of π from the series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots$$

Write a C program to convert a binary number into a decimal number.

$$5+5=10$$

- (h) Write the differences between 'while' loop and 'do-while' loop using examples. Write a C program to check whether the given number is an Armstrong number. (An Armstrong number is one that is equal to the sum of cubes of individual digits. For

$$\text{ex. } 153 = 1^3 + 5^3 + 3^3 \qquad 5+5=10$$