

Total number of printed pages-7

3 (Sem-4/CBCS) MAT HC 2

2022

**MATHEMATICS**

(Honours)

Paper : MAT-HC-4026

**(Numerical Methods)**

Full Marks : 60

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer **any seven** questions :  $1 \times 7 = 7$

- (a) What do you mean by an algorithm ?
- (b) What is the underlying theorem of bisection method ?
- (c) Write the iterative formula of secant method for solving an equation  $f(x) = 0$ .
- (d) Consider the system of equations  $Ax = b$ . In which method, the matrix  $A$  can be decomposed into the product of two triangular matrices ?

Contd.



- (e) Name one iterative method for solving a system of linear equations.
- (f) Write the iterative formula of Newton-Raphson method to find the square root of 15.
- (g) What do you mean by interpolating polynomial?
- (h) Show that  $\Delta = E - 1$ .
- (i) What do you mean by numerical differentiation?
- (j) Write the formula for second order central difference approximation to the first derivative.

2. Answer **any four** questions :  $2 \times 4 = 8$

- (a) Examine whether the fixed point iteration method is applicable for finding the root of the equation :

$$2x = \sin x + 5.$$

- (b) Define rate of convergence and order of convergence of a sequence.

- (c) Prove that  $\mu = \left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$  where  $\mu$  and  $\delta$  are average and central difference operators.

- (d) Verify that the following equation has a root on the interval  $(0, 1)$  :

$$f(x) = \ln(1+x) - \cos x = 0.$$

- (e) If  $P_1(x) = a_0 + a_1x$  such that  $P_1(x_0) = f_0$  and  $P_1(x_1) = f_1$ , then obtain an expression for  $P_1(x)$  in terms of  $x_i$ 's and  $f_i$ 's ( $i = 0, 1$ ).

- (f) Show that  $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$ .

- (g) What do you mean by degree of precision of a quadrature rule? If a quadrature rule  $I_n(f)$  integrates  $1, x, x^2$  and  $x^3$  exactly, but fails to integrate  $x^4$  exactly, then what will be the degree of precision of  $I_n(f)$ ?

- (h) Mention briefly about the use of Euler's method.

3. Answer **any three** questions :  $5 \times 3 = 15$

- (a) Give a brief sketch of the method of false position.
- (b) Give the geometrical interpretation of Newton-Raphson method.



(c) Construct an algorithm for the secant method.

(d) Show that an  $LU$  decomposition is unique up to scaling by a diagonal matrix.

(e) Discuss about the advantages and disadvantages of Lagrange's form of interpolating polynomial.

(f) Given  $f(2)=4$ ,  $f(2.5)=5.5$ , find the linear interpolating polynomial using Lagrange's interpolation. Hence find an approximate value of  $f(2.2)$ .

(g) Derive the closed Newton-Cotes quadrature formula corresponding to  $n=1$ . Why is this formula called trapezoidal rule?

(h) Evaluate  $\int_0^1 \tan^{-1} x \, dx$  using Simpson's  $\frac{1}{3}$ rd rule.

4. Answer **any three** questions:  $10 \times 3 = 30$

(a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation :

$$f(x) = x^3 - 5x + 1 = 0.$$

(b) Apply Newton-Raphson method to determine a root of the equation :

$$f(x) = \cos x - xe^x = 0.$$

Taking the initial approximation as  $x_0 = 1$ , perform five iterations.

(c) Form an  $LU$  decomposition of the following matrix :

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{pmatrix}$$

(d) Find the order of convergence of the iterative method  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$  to

compute an approximation to the square root of a positive real number  $a$ . To find the real root of  $x^3 - x - 1 = 0$  near  $x = 1$ , which of the following iteration functions give convergent sequences?

(i)  $x = x^3 - 1$

(ii)  $x = \frac{x+1}{x^2}$



- (e) Construct the difference table for the sequence of values :

$$f(x) = (0, 0, 0, \varepsilon, 0, 0, 0).$$

where  $\varepsilon$  is an error. Also show that —

- (i) the error spreads and increases in magnitude as the order of differences is increased;
  - (ii) the errors in each column have binomial coefficients.
- (f) Let  $x_0 = -3$ ,  $x_1 = 0$ ,  $x_2 = e$  and  $x_3 = \Pi$ . Determine formulas for the Lagrange's polynomials  $L_{3,0}(x)$ ,  $L_{3,1}(x)$ ,  $L_{3,2}(x)$  and  $L_{3,3}(x)$  associated with the given interpolating points.
- (g) For the function  $f(x) = \ln x$ , approximate  $f'(3)$  using —
- (i) first order forward difference, and
  - (ii) first order backward difference approximation formulas.

[Starting with step size  $h = 1$ , reduce it by  $\frac{1}{10}$  in each step until convergence.]

$$5+5=10$$

- (h) Solve the initial value problem :

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \leq t \leq 2.5$$

$$x(1) = 1,$$

using Euler's method with step size  $h = 0.5$  and find an approximate value of  $x(2.5)$ .