3 (Sem – 6 / CBCS) MAT HC 2

2022

MATHEMATICS

(Honours)

Paper : MAT–HC–6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1.   Answer *any seven* :   1×7=7

  (i)   The equation of the form

    $P_p + Q_q = \mathbb{R}$ is known as

    (a)   Charpit's equation

    (b)   Lagrange's equation

    (c)   Bernoulli's equation

    (d)   Clairaut's equation

            (Choose the correct answer)

(ii) How many minimum no. of independent variables does a partial differential equation require ?

(iii) Find the degree and order of the equation

$$\frac{\partial^3 z}{\partial x^3} + \left(\frac{\partial^3 z}{\partial x \partial y^2}\right)^2 + \frac{\partial z}{\partial y} = sin(x + 2y)$$

(iv) Which method can be used for finding the complete solution of a non-linear partial differential equation of first order

(a) Jacobi method

(b) Charpit's method

(c) Both *(a)* and *(b)*

(d) None of the above

(Choose the correct answer).

(v) State *True* **Or** *False* :

The equation

$$u_{xx} + u_{yy} + u_{zz} = 0$$

is an Hyperbolic equation.

(vi) Fill in the blanks :

$$\left(\frac{\partial z}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + z = 0$$

is a _____ _____ order partial differential equation.

(vii) The characteristic equation of $y u_x + x u_y = u$ is

(a) $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{du}{u}$

(b) $\dfrac{dx}{y} = \dfrac{dy}{x} = \dfrac{du}{u}$

(c) $\dfrac{dx}{u} = \dfrac{dy}{x} = \dfrac{du}{y}$

(d) None of the above

(Choose the correct answer)

(viii) State *True* **Or** *False*

$x u_x + y u_y = u^2 + x^2$ is a semi-linear partial differential equation.

(ix) Fill in the blanks :

A solution $z = z(x, y)$ when interpreted as a surface in 3-dimensional space is called _____.

(x) The partial differential equation is elliptical if

(a) $B^2 - 4AC > 0$

(b) $B^2 - 4AC \geq 0$

(c) $B^2 - 4AC \leq 0$

(d) $B^2 - 4AC < 0$

(Choose the correct answer)

2. Answer **any four** : 2×4=8

   (i) Define quasi-linear partial differential equation and give *one* example.

   (ii) Show that a family of spheres
$$(x-a)^2 + (y-b)^2 = r^2$$ satisfies the partial differential equation
$$z^2(p^2 + q^2 + 1) = r^2$$

   (iii) Eliminate the constants a and b from
$$z = (x+a)(y+b).$$

   (iv) Determine whether the given equation is hyperbolic, parabolic or elliptic
$$u_{xx} - 2u_{yy} = 0.$$

   (v) Solve the differential equation $p + q = 1$.

   (vi) Explain the essential features of the "Method of separation of variables".

   (vii) Mention when Charpit's method is used. Name a disadvantage of Charpit's method.

   (viii) What is the classification of the equation
$$u_{xx} - 4u_{xy} + 4u_{yy} = e^y$$

3. Solve **any three** : 5×3=15

   (i) Form a partial differential equation by eliminating arbitrary functions f and F from $y = f(x-at) + F(x+at)$.

   (ii) Solve
$$y^2 p - xyq = x(z - 2y)$$

   (iii) Find the integral surface of the linear partial differential equation
$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$
which contains the straight line
$$x + y = 0, \quad z = 1.$$

   (iv) Find the solution of the equation $z = pq$ which passes through the parabola
$$x = 0, \quad y^2 = z.$$

   (v) Find a complete integral of the equation
$$x^2 p^2 + y^2 q^2 = 1.$$

   (vi) Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution.

(vii) Apply the method of separation of variables $u(x, y) = f(x) g(y)$ to solve the equation $u_x + u = u_y$,

$$u(x, 0) = 4e^{-3x}.$$

(viii) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

4. Answer *any three* : 10×3=30

(i) Solve $\left(p^2 + q^2\right)y - qz = 0$ by Jacobi method.

(ii) Solve $z^2 = pqxy$ by Charpit's method.

(iii) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$$

(iv) Solve

$$(mz - ny)p + (nx - lz)q = ly - mx$$

(v) Use $v = \ln u$ and $v = f(x) + g(y)$ to solve the equation

$$x^2 u_x^2 + y^2 u_y^2 = u^2.$$

(vi) Find the solution of the equation

$$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$$

which passes through the x axis.

(vii) Find the canonical form of the equation

$$y^2 u_{xx} - x^2 u_{yy} = 0.$$

(viii) Classify the second order linear partial differential equation with example.