3 (Sem–6/CBCS) MAT HE 1/2/3/4

2022

MATHEMATICS

(Honours Elective)

DSE (H) – 3

Answer the Questions from any one Option.

OPTION – A

(Boolean Algebra and Automata Theory)

Paper : MAT–HE–6016

Full Marks : 80

Time : Three hours

OPTION – B

(Biomathematics)

Paper : MAT–HE–6026

Full Marks : 80

Time : Three hours

OPTION – C

(Mathematical Modeling)

Paper : MAT–HE–6036

Full Marks : 60

Time : Three hours

OPTION – D

(Hydromechanics)

Paper : MAT–HE–6046

Full Marks : 80

Time : Three hours

*The figures in the margin indicate
full marks for the questions.*

OPTION – A

(Boolean Algebra and Automata Theory)

Paper : MAT–HE–6016

1. Answer **any ten** questions from the following : $1 \times 10 = 10$

    (a) Let P be a ordered set equipped with an order relation $\leq$. Then which of the following is symmetric condition ?

        (i) $x \leq x$

        (ii) $x \leq y$ and $y \leq x$ imply $x = y$

        (iii) $x \leq y$ and $y \leq x$ imply $x \leq z$

        (iv) None of the above

             (Choose the correct answer)

    (b) The symbol $x \| y$ is used for

        (i) $x \leq y$ and $y \nleq x$

        (ii) $x \nleq y$ and $y \leq x$

        (iii) $x \nleq y$ and $y \nleq x$

        (iv) None of the above

             (Choose the correct answer)

    (c) Let P and Q be ordered set. A map $\phi : P \to Q$ is said to be ordered preserving, if

        (i) $x \leq y$ in P imply $\phi(x) \leq \phi(y)$ in Q

        (ii) $x \leq y$ in $P \Leftrightarrow \phi(x) \leq \phi(y)$ in Q

        (iii) it is an ordered embedding which map P onto Q

        (iv) None of the above

             (Choose the correct answer)

    (d) Let P be a nonempty ordered set. P is called complete lattice, if

        (i) $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$

        (ii) $\vee S$ and $\wedge S$ exist for all $S \subseteq P$

        (iii) $S \vee P$ and $S \wedge P$ exist for all $P \subseteq S$

        (iv) None of the above

             (Choose the correct answer)

    (e) For a given lattice L and $a, b \in L$ which is true ?

        (i) $a \wedge b = \sup \{a, b\}$

        (ii) $a \vee b = \sup \{a, b\}$

        (iii) $a \vee b = \inf \{a, b\}$

        (iv) None of the above

             (Choose the correct answer)

(f) If $(B, +, \cdot, ')$ is a Boolean algebra and $a \in B$, then

(i) $a' + a = 0, \ aa' = 1$

(ii) $a + 1 = 0, \ a.0 = 1$

(iii) $a' + a = 1, \ aa' = 0$

(iv) None of the above

(Choose the correct answer)

(g) What is duality principle of ordered set?

(h) What is maximal element of an ordered set?

(i) Define sublattice of a lattice.

(j) What is context-free grammar?

(k) Write the full form of PDA in automata theory.

(l) Define Boolean algebra in terms of lattice.

(m) Write De Morgan's law in Boolean algebra $(B, +, \cdot, ')$.

(n) What is disjuctive normal form for a Boolean expression?

(o) Write idempotent law for Boolean algebra.

2. Answer **any five** from the following:

$2 \times 5 = 10$

(a) Let P be a lattice. Then for all $a, b, c \in P$, prove that

$$a \leq b \Rightarrow a \vee c \leq b \vee c$$

(b) Let $B = \{2, 3, 4, 5, 6, 12, 36, 48\}$ and S be the relation 1, 'divides' on B. Draw the Hasse diagram of S.

(c) Write truth table for '$\rightarrow$' operator.

(d) State the principle of duality for Boolean algebra.

(e) Write dual of $a \vee \overline{(\overline{b} \vee a \wedge b)} = 1$.

(f) Consider the following language over $A = \{a, b\} \ L = \{a^m b^n, \ m > 0, \ n > 0\}$. Find a regular expression S over $A = \{a, b\}$ such that $L_i = L(x)$ for $i = 1, 2, 3$.

(g) Prove that every singleton of a lattice L is a sublattice of L.

(h) Prove that in a lattice L,

$$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$$

for all $x, y, z \in L$.

3. Answer **any four** questions from the following : 5×4=20

(a) Prove that elements of an arbitrary lattice satisfy the following distributive inequality :

$$x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$$

$$x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$$

(b) Show that the language

$$L = \{a^m b^m ; m \text{ positive}\} \text{ is not regular.}$$

(c) Find the language $L(G)$ over $\{a, b, c\}$ generated by the grammar G with production

$$S \rightarrow aSb, \quad aS \rightarrow Aa, \quad Aab \rightarrow c.$$

(d) Consider the CFG G defined by production $S \rightarrow aS \,|\, Sb \,|\, a \,|\, b$.

(i) Prove by induction on the string length that no string $L(S)$ has *ba* as *a* substring

(ii) Describe $L(G)$ informally. Justify your answer using part (i)

(e) Find a Boolean polynomial p that induces the function f :

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

(f) Construct a grammar for the language

$$L = \{a^x b^y, x > y > 0\}$$

(g) Suppose that L is any language not necessarily regular, whose alphabet is $\{0\}$, i.e., the strings of L consist of 0's only. Prove that L^* is regular.

(h) Express the Boolean expression $x' + yz$ in conjunctive normal form in the variables x, y, z.

4. Answer **any four** from the following:

$$10 \times 4 = 40$$

(a) Determine the minimal form of p, which is given in DNF:

$$p = v'w'x'y'z' + v'w'x'yz' + v'w'xy'z' + v'w'xyz'$$

$$+ v'wx'y'z + v'wx'yz' + v'wxy'z + v'wxyz'$$

$$+ v'wxyz + vw'x'y'z' + v'w'x'y'z + vw'xy'z$$

$$+ vwx'yz' + vwxy'z' + vwxyz' + vwxyz$$

(b) (i) Let $(L, \le)$ be a lattice ordered set. If we define $x \wedge y = \inf(x, y)$, $x \vee y = \sup(x, y)$, then prove that $(L, \wedge, \vee)$ is an algebraic lattice.

(ii) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define $x \le y \Leftrightarrow x \wedge y = x$, then prove that $(L, \le)$ is a lattice ordered set.

(c) Let $G = (V, T, P, S)$ be a CFG. If the recursive inference procedure tells us that terminal string a is in the language of variable A, then prove that there is a parse tree with root A and yield w.

(d) Let $G = (V, T, P, S)$ be a CFG and suppose there is a parse tree with root labeled by variable A and with yield w, where w is in T^*, then prove that there is a leftmost derivation $A \underset{lm}{\overset{*}{\Rightarrow}} w$ in grammar G.

(e) Obtain a Boolean expression which represents the following circuit:
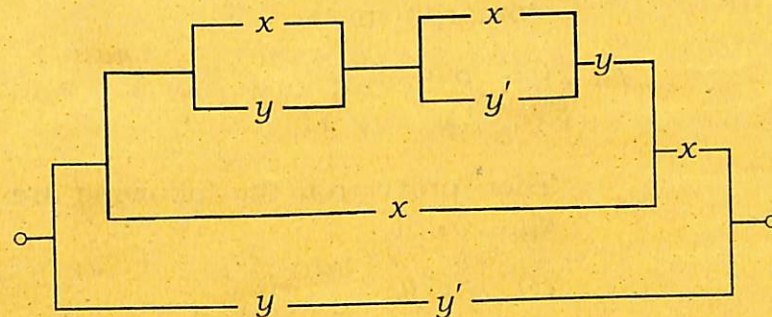
Draw an equivalent circuit as simple as possible. Show that current will flow through the circuit if and only if the switch x is on.

(f) Let $f : B^n \to B$ be a function such that $f \ne 0$. Then prove that there exists a Boolean expression α in DNF such that $f(b_1, b_2, \cdots, b_n) = \alpha(b_1, b_2, \cdots, b_n)$ for all $(b_1, b_2, \cdots, b_n) \in B^n$.

(g) (i) Let $X = \{1, 2, \cdots, n\}$ and define

$\phi : Q(X) \rightarrow 2^n$ by

$\phi(A) = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n\}$ where

$$\varepsilon_i = \begin{cases} 1 & \text{if} \quad i \in A \\ 0 & \text{if} \quad i \notin A \end{cases}$$

Then prove that ϕ is an order-isomorphism.

(ii) Let P be an ordered set and $x, y \in P$.

Then prove that the following are equivalent :

(1) $x \le y$

(2) $\downarrow x \subseteq \downarrow y$

(3) $(\forall Q \in Q(P)) y \in Q \Rightarrow x \in Q$

(h) Let L be a lattice. Then prove that the following are equivalent :

(i) L is nonmodular if and only if $N_5 \rightarrow L$

(ii) L is nondistributive if and only if $N_5 \rightarrow L$ or $M_5 \rightarrow L$

(i) Let $f : B \rightarrow C$ where B and C are Boolean algebras.

(i) Assume f is a lattice homomorphism. Then prove that the following are equivalent :

(1) $f(0) = 0$ and $f(1) = 1$

(2) $f(a') = (f(a))'$ for all $a \in B$

(ii) If f preserves $'$, then prove that f preserves $\vee$ if and only if f preserves $\wedge$.

(j) In a Boolean algebra $(B, +, \cdot, ')$, for $a, b, c \in B$, prove that

(i) if $b + a = c + a$ and $b + a' = c + a'$, then $b = c$

(ii) $a + (b + c) = (a + b) + c$

and

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

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OPTION – B

Paper : MAT–HE–6026
(Biomathematics)

1. Answer **any ten** questions :   1×10=10

   (a) What is a difference equation ?

   (b) Write a biological factor that to be the cause of density dependence.

   (c) When is a mathematical model called autonomous ?

   (d) Give an example of a nonlinear mathematical model.

   (e) Define equilibrium value of a model $\Delta P = G(P_t)$.

   (f) Write the order of the following partial differential equation :

   $$u_{xxy} + 2 x u_{xy} + 8u = 7y$$

   (g) State whether the following partial differential equation is linear, quasilinear or non-linear :

   $$u_x u_{xx} + x u u_y = \sin y$$

   (h) Write a difference between continuous growth and discrete growth.

   (i) Write the condition for local stability for a system defined by

   $$f(x) = r\left(1 - \frac{x}{k}\right) x$$

   (j) Fill in the blank :
   Product of characteristic roots or eigen-vectors of a matrix A is equal to _____.

   (k) What is the central idea of a Lyapunov method ?

   (l) What is bifurcation ?

   (m) If we flip a fair coin what is the probability of outcome 'head' ?

   (n) Write the *two* extreme equilibrium states in a heart beat cycle.

   (o) A Jukes-Cantor model is one-parameter model of mutation.   (True/False)

2. Answer **any five** :   2×5=10

   (a) Explain why the model $\Delta Q = r Q$ cannot be biologically meaningful for describing a population when $r < -1$, where r is the per capita growth rate of quantity Q.

   (b) Write *two* basic requirements of a mathematical problem to be properly posed.
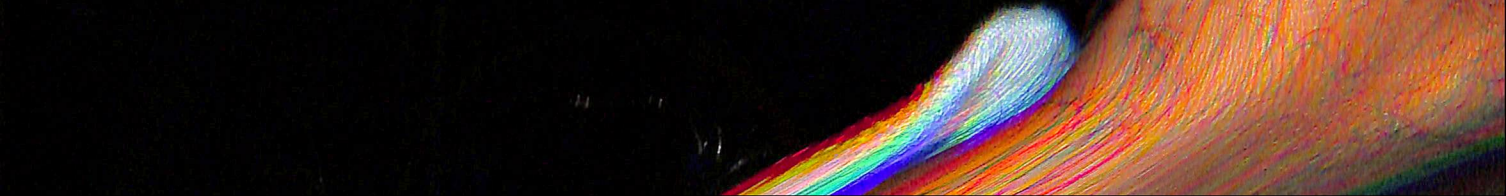
(c) State the existence theorem.

(d) Prove that, the sum of characteristic roots or eigenvalues of a square matrix A of order 2 is equal to trace of A.

(e) With the help of heart beat cycle, discuss briefly about the threshold effect.

(f) What is the difference between a Jukes-Cantor model and Kimura model ?

(g) Write *two* additional assumptions to basic Markov model by Jukes-Cantor model.

(h) What are Mendel's isolated seven characteristics for the pea plant ?

3. Answer *any four* : 5×4=20

(a) In the early stages of the development of a frog embryo, cell division occurs at a fairly regular rate. Suppose you observe that all cells divide and hence the number of cells double, roughly every half hour.

Write down an equation modeling this situation. You should specify how much real world time is represented by an increament of I in t and what the initial numbers of cell is.

(b) Prove that if a model
$$P_{t+1} = F(P_t)$$
has equilibrium P^* then $\left| F'(P^*) \right| > 1$ implies P^* is unstable, while $\left| F'(P^*) \right| < 1$ implies P^* is stable.

(c) Suppose a virus enters the blood stream and develops at a rate proportional to its concentration. An antidote to the virus is administered at a time h and decays according to the law that the rate of reduction of concentration of drug to be proportional to its concentration. Write down a simple model of virus-antidote interaction and estimate the dosage of antidote needed to eradicate the viral infection as quickly as possible.

(d) Determine the solution of
$$u_{xx} - u_{yy} = 1$$
$$u(x, 0) = \sin x$$
$$u_y(x, 0) = x$$

(e) Consider the nonlinear conservative system in which $\mu \geq 0$ and
$$\dot{x} = y, \quad \dot{y} = \mu \sin x - x$$
Examine the nature of equilibrium.

(f) Suppose a family has 6 children. What is the probability that 4 are boys and 2 are girls ?

(g) Consider the space clamp model where the induced current I is assumed constant. Determine the critical points of this system when the current takes the value

$$I = \frac{b}{\gamma}, \quad I = \frac{ab}{\gamma}.$$

(h) What is phytogenetic tree ? Why is it reasonable to assume that evolutionary relationships can be modified by drawing trees ? What would it mean if there were a loop of edges ?

4. Answer **any four** : 10×4=40

(a) Develope and analyse a growth and decay model through ordinary differential equations.

(b) Develope and analyse a nonlinear model through ordinary differential equations.

(c) Using a suitable Lyapunov function, explain the stability condition of a prey-predator model.

(d) Analyse and develop a simple epidemic model by using ordinary differential equation.

(e) Provide a full phase plane analysis for a heart beat equation.

(f) Discuss briefly the stability of a limit cycle.

(g) Suppose a population of cells grows logistically according to the law of logistic growth, with a per capita growth rate of $N_o = 0.1h^{-1}$ (a small population that is not approaching carrying capacity will take $10h$ to increase by a factor of e). If the initial size of the population is 10% of the carrying capacity, how long will it take for the population to reach 95% of the carrying capacity ?

(h) Explain briefly about Poincare plane.

(i) Write a brief note on the development of logistic growth model.

(j) Write a brief note on bifurcation and chaos with a suitable example.

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OPTION – C

(Mathematical Modelling)

Paper : MAT–HE–6036

1. Answer **any seven** questions : 1×7=7

(a) Define a power series.

(b) Write the condition that the Taylor series with centre $x = a$ becomes the Maclaurin series.

(c) Does a differential equation with singular point at zero only have power series of the form

$$y(x) = \Sigma C_n x^n \ ?$$

(d) Write the value of $\left\lfloor \dfrac{1}{2} \right.$.

(e) When does an improper integral over an infinite interval converge ?

(f) Does a Frobenius series always represent a power series ?

(g) Write the value of

$$L\{t\} = \int_0^\infty e^{-st}\, dt$$

(h) What is the value of $u(t)$, when $t \geq 0$ in a unit step function ?

(i) Write the condition that a tangent series for $\tan x$ is convergent.

(j) Choose the correct answer :

A Monte Carlo simulation is a

(i) probabilistic model

(ii) deterministic model

(iii) discrete model

(iv) None of the above

2. Answer **any four** questions : 2×4=8

(a) Calculate $\left\lfloor \dfrac{5}{2} \right.$

(b) Show that

$$L\{t\cos(kt)\} = \frac{s^2 - k^2}{\left\{ s^2 + k^2 \right\}^2}$$

(c) Prove the linearity property of Laplace transform.

(d) Find the inverse Laplace transform of

$$G(s) = \frac{1}{s^2(s-a)}.$$

(e) Prove that

$$\left\lceil n+1 \right. = \left\lfloor n \right.$$

(f) Find the inverse Laplace transform of

$$F(s) = \frac{3}{2s - 4}$$

(g) Define a linear congruence.

(h) Explain briefly a single server queuing model.

3. Answer **any three** questions :   5×3=15

(a) Solve the equation
$$x^2 y' = y - x - 1$$

(b) Investigate the value at the point $x = 0$ for the differential equation
$$x^4 y'' + \left(x^2 \sin x\right) y' + (1 - \cos x) y = 0$$

(c) Solve the initial value problem
$$x^4 - x' - 6x = 0$$
$$x(0) = 2$$
$$x'(0) = -1$$

(d) Show that
$$L\left\{te^{at}\right\} = \frac{1}{(s-a)^2}$$

(e) Use Laplace transformation to solve the initial value problem
$$x'' + w_0^2 x - F_0 \sin wt$$
$$x(0) = 0 = x'(0)$$

(f) Find the Frobenius solution for Bessel's equation of order zero
$$x^2 y'' + xy' + x^2 y = 0$$

(g) Using Monte Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle
$$Q : x^2 + y^2 = 1, \; x \geq 0, \; y \geq 0$$
Use the equation $\dfrac{\pi}{4} = $ area $Q/$area S.

(h) Maximize $x + y$

subject to $x + y \leq 6$
$$3x - y \leq 9$$
$$x, y \geq 0$$

4. Answer **any three** parts :   10×3=30

(a) Find the general solution in powers of x of the equation
$$\left(x^2 - 4\right) y'' + 3xy' + y = 0$$
Then find the particular solution with
$$y(0) = 4, \; y'(0) = 1.$$

(b) Determine the radius of convergence of a series solution of

$$\left(x^2 + 9\right)y'' + xy' + x^2 y = 0$$

in powers of x and in powers of $(x-4)$ by finding the singularities.

(c) Find linearly independent solutions of

$$y'' - xy' - x^2 y = 0$$

(d) Find Frobenius series solutions of

$$2x^2 y'' + 3xy' - \left(x^2 + 1\right)y = 0 .$$

(e) Using Monte Carlo simulation write an algorithm to calculate the volume of a sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant $x \geq 0, y \geq 0, z \geq 0$.

(f) Using simplex method optimize $6x + 4y$

subject to $x + y \leq 12$

$$x + y \leq 24$$
$$2x + 5y < 80, \quad x_1 y \geq 0$$

(g) Explain briefly the middle square method.

Use this method to generate

(i) 10 random numbers using $x_0 = 1009$

(ii) 15 random numbers using $x_0 = 3043$

(h) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between Successive ships (in minutes)	20	30	15	120	25
Unloading time	55	45	60	75	80

(i) Draw the time-line diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time.

(ii) List the waiting time for all the ships and find the average waiting time.

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OPTION – D

Paper : MAT–HE–6046

(Hydromechanics)

1. Answer the following questions : *(any ten)*

   $1 \times 10 = 10$

   (a) State the property for which a mass of liquid can be used as a 'machine' for the purpose of multiplying power.

   (b) Give example of an elastic fluid.

   (c) Define centre of pressure of a plane area immersed in a fluid.

   (d) Name the instrument used to measure atmospheric pressure.

   (e) Flow elasticity of a fluid is measured ?

   (f) Define absolute zero of temperature.

   (g) What is meant by thermal capacity of a body ?

   (h) Name the *two* methods of treating the general problem of hydrodynamics.

   (i) What is tube of flow ?

   (j) Define a perfect fluid with example.

   (k) If $\bar{q}$ be the velocity field of an incompressible homogeneous fluid, what is the value of $\nabla \cdot \bar{q}$?

   (l) What is an adiabatic change ?

   (m) State Boyle's law.

   (n) What is the physical significance of the equation of continuity ?

   (o) For what type of motion of a particle, the stream lines and path lines coincidie ?

2. Answer *any five* of the following : $2 \times 5 = 10$

   (a) Obtain the differential equations of the lines of force at any point.

   (b) Prove that the pressure at any point of a fluid varies as the depth of the point from the surface when there is no atmosphere.

   (c) "The centre of pressure of any plane area, not horizontal, is below its centroid." Explain why.

(d) Find the equations of the lines of force for a fluid motion in which

$$u = -\frac{c^2 y}{r^2}, \quad v = -\frac{c^2 x}{r^2}, \quad w = 0$$

where r denotes the distance from z-axis.

(e) What is meant by velocity potential ? Does velocity potential exist for a rotational flow ?

(f) The velocity $\bar{q}$ in the three-dimensional flow field for an incompressible fluid is given by $\bar{q} = \lambda x\hat{i} - y\hat{j} - z\hat{k}$. Find the value of λ.

(g) State the relations among pressure, density and temperature for a given mass of gas under different conditions.

(h) Find the resultant vertical pressure on any surface of a homogeneous liquid at rest under the action of gravity.

3. Answer **any four** of the following : 5×4=20

(a) A closed tube in the form of an ellipse with its major axis vertical is filled with three different liquids of densities ρ_1, ρ_2, ρ_3 respectively. If the distances of the surfaces of separation from either focus be r_1, r_2, r_3 respectively, prove that $r_1 (\rho_2 - \rho_3) + r_2 (\rho_3 - \rho_1) + r_3 (\rho_1 - \rho_2) = 0$.

(b) A given volume of liquid is at rest on a fixed plane under the action of a force, to a fixed point in the plane, varying as the distance. Find the pressure at any point of the fluid and the whole pressure on the fixed plane.

(c) A mass of homogeneous liquid, contained in a vessel, revolves uniformly about a vertical axis. Determine the pressure at any point and the surfaces of equal pressure.

(d) A circular area of radius a is immersed with its plain vertical and centre at a depth h. Find the depth of centre of pressure below the free surface.

(e) If the law connecting the pressure and density of the air were $p = k\rho^n$, prove that neglecting variations of gravity and temperature, the height of the atmosphere would be $\frac{n}{n-1}$ times the homogeneous atmosphere.

(f) Given the velocity field

$$\bar{q} = Ax^2 y\hat{i} + By^2 z\hat{j} + Czt^2 \hat{k}$$

Determine the acceleration of the fluid particle.

(g) A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \theta}(\rho_\omega) = 0$$

where ω is the angular velocity of a particle whose azimuthal angle is θ at time t.

(h) A vertical rectangle is exposed to the action of the atmosphere at a constant temperature. Assuming that the pressure exerted by the atmosphere varies with the height according to the usual law, determine the position of the centre of pressure of the rectangle.

4. Answer **any four** of the following : 10×4=40

(a) If a mass of fluid is at rest under the action of given forces, obtain the equation which determines the pressure at any point of the fluid.

Also deduce a necessary condition of equilibrium.   7+3=10

(b) A liquid of given volume V is at rest under the forces $X = \dfrac{-\mu x}{a^2}$, $Y = \dfrac{-\mu y}{b^2}$, $Z = \dfrac{-\mu z}{c^2}$. Find the pressure at any point. Also find the equation to the free surface and the surfaces of equal pressures.   5+2+3=10

(c) A solid octant of a sphere is immersed with one plane face in the surface. Prove that the resultant pressure on the curved surface reduces to a single force of magnitude $\dfrac{1}{6}\rho \pi a^3 \sqrt{\pi^2 + 8}$

Also show that the equation of the line of action of single resultant is $x = y = \dfrac{2z}{\pi}$.   8+2=10

(d) Obtain the equation of continuity for a fluid in motion, in the form

$$\frac{\partial \rho}{\partial t} + div(\rho \bar{q}) = 0$$

where ρ and $\bar{q}$ are respectively density and the velocity of the fluid. Deduce the form of equation of continuity when the fluid is homogeneous and incompressible.   8+2=10

(e) Define stream lines and ·path lines. Determine the stream lines and the path of the particles, when the velocity field

is given by $u = \dfrac{x}{1+t}$, $v = \dfrac{y}{1+t}$, $w = \dfrac{z}{1+t}$.

4+6=10

(f) Assuming that the temperature of the atmosphere is uniform, show that as the altitude increases in A.P., the pressure decreases in G.P.

Hence deduce a formula for comparing the differences of level of two stations above earth's surface by a barometer. What changes have to be applied if the temperature is not uniform but acceleration due to gravity is constant ?

6+2+2=10

(g) (i) Two volumes v_1 and v_2 of different gases at pressures p_1 and p_2 and absolute temperatures T_1 and T_2 are mixed together, so that the volume of the mixture is V and absolute temperature if T. Prove that the pressure of the mixture is

$$\frac{T}{V}\left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right)$$

(ii) Masses m and m' of two gases in which the ratio of the pressure to the density are respectively k and k', are mixed at the same temperature. Prove that the ratio of the pressure to the density in the

compound is $\dfrac{mk + m'k'}{m + m'}$.

5+5=10

(h) Obtain the differential equations of the curves of equal pressure and density for a fluid at rest under the action of the forces x, y, z per unit mass. If $X = y(y+z)$, $Y = z(z+x)$, $Z = y(y-x)$, prove that surfaces of equal pressures are hyperbolic paraboloids

$y(x+z) = c(y+z)$ and the curves of equal pressure and density are given by $y(x+z) = $ constant, $y+z = $ constant.

5+5=10

(i) Define the following : 4+6=10

(i) Steady and unsteady flow

(ii) Rotational and irrotational motion

Show that $u = -\dfrac{2xyz}{\left(x^2 + y^2\right)^2}$,

$v = -\dfrac{\left(x^2 - y^2\right)z}{\left(x^2 + y^2\right)^2}$, $w = \dfrac{y}{x^2 + y^2}$

are the velocity components of a possible liquid motion.

(j) A given mass of air is contained within a closed air-tight cylinder with its axis vertical. The air is rotating in relative equilibrium about the axis of the cylinder. The pressure at the highest point of its curved surface is p, and the pressure at the highest point of its axis is P. Prove that, if the fluid were absolutely at rest, the pressure at the upper end of the axis would be

$\dfrac{p - P}{\log p - \log P}$, where the weight of the air

is taken into account.

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