3 (Sem-1) MAT M 1

Lib. Set. 2.

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper: 1.1

(Algebra and Trigonometry)

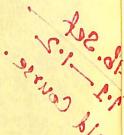
Full Marks: 80

Time: Three hours

The figures in the margin indicate full marks for the questions.

- 1. Answer the following questions: 1×10=10
 - (a) Give an example of a relation on the set of real numbers R which is reflexive and transitive but not symmetric.
 - (b) Is generator of a cyclic group always unique?
 - (c) Define Hermitian matrix.
 - (d) Find all partitions of the set $x = \{1, 2, 3\}$.

- (e) Find the value of i^i .
- (f) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{pmatrix}$



- (g) Examine whether the inverse of the matrix $\begin{pmatrix} 1 & w \\ w & w^2 \end{pmatrix}$ exists or not.
- (h) Define an operation * on the set of real numbers R where $a*b=a+2b, \forall a,b \in R$
- (i) What is normal form of a matrix?
- (j) Find the amplitude of the complex number -1-i.
- 2. Give the answer of the following: $2 \times 5 = 10$
 - (a) Can a non-Abelian group have an Abelian subgroup? Justify your answer.

- (b) If $f: A \to B$ and $g: B \to C$ are bijective mappings, then prove that $g \circ f$ is also a bijective mapping.
- (c) Prove that $\pi = 2\sqrt{3} \left(1 \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} \frac{1}{7 \cdot 3^3} + \dots \right)$

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- (d) Solve the equation $x^3 + 6x + 20 = 0$ if one root is 1+3i.
- (e) Show that the relation defined on $N \times N$ by $(a,b) \sim (c,d)$ iff a+d=b+c is an equivalence relation.
- 3. Answer *any four*: 5×4=20
 - (a) Define an equivalence relation on a nonempty set. Show that the relation 'congruence modulo m' is an equivalence relation on the set of integers. 1+4=5

3

- (b) Let $f: A \to B$, $g: B \to C$, $h: C \to D$ be three mappings. Prove that
 - (i) $h \circ (g \circ f) = (h \circ g) \circ f$
 - (ii) $f \circ i = f$ and $j \circ f = f$ where $i: A \rightarrow A$ and $j: B \rightarrow B$ are identity mappings.
- (c) If the matrices A and B are commute, then show that A^{-1} and B^{-1} are also commute.
- (d) Prove that every group of prime order is cyclic.
- (e) Solve $x^4 2x^3 21x^2 + 22x + 40 = 0$ whose roots are in AP.
- (f) Test the consistency and solve:

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

4. Answer any two:

- 5×2=10
- (a) If α , β , γ are the roots of the equation $x^3 px^2 + qx r = 0 \text{ then find the value}$ of $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ in terms of p, q and r.
- (b) Find the condition that the cubic $x^3 px^2 + qx r = 0$ should have its roots in harmonic progression.
 - (c) If $f: A \to B$ and $g: B \to C$ be one-one and onto maps, then show that $g \circ f$ is inversible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
- 5. Answer any two:

- $5 \times 2 = 10$
- (a) Prove that the order of a cyclic group is equal to the order of any generator of the group.
- (b) Prove that every finite group G is isomorphic to a permutation group.

5

- (c) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$, prove that $\alpha^2 \operatorname{sec} h^2 \phi + \beta^2 \operatorname{cosec} h^2 \phi = 1.$
- Answer any two: in terms of p, q and p

(a) Prove that

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$$

 $x^3 - px^2 + qx - r = 0$ then find the value

- px2+qx-r=0 should have its (b) Solve $x^3 - 3x - 1 = 0$ by Cardon's method.
 - (c) If H is a subgroup of G, then prove that there is a one to one correspondence between set of left coset of H in G and the set of right coset of H in G.
- Answer any two: (a) Prove that the order of a cyclic group is

$$5 \times 2 = 10$$

5. Answer any two:

(a) If $tan(\theta + i\phi) = cos\alpha + i sin\alpha$, prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4}$$
 and $\phi = \frac{1}{2} \log_a \tan\left(\frac{\pi}{4} + \frac{\lambda}{2}\right)$

- Prove that the necessary and sufficient condition for a matrix A to possess an inverse is that $|A| \neq 0$.
- Prove that every square matrix satisfies (c) its own characteristic equation.