Total number of printed pages–7

3 (Sem–1) MAT M 2

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

(a) What is the nth derivative of $sin(ax+b)$?

Contd.

(b) If $Z = x^3 y^4 f(y/x)$, then find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

(c) Write the polar subtangent for the curve $r = a\theta$.

(d) Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

(e) Define the curvature of a curve at point on it.

(f) What is the value of $\int_0^{\pi} \frac{\sin 4x}{\sin x} dx$?

(g) Evaluate $\int_{-\pi/2}^{\pi/2} \cos x \, dx$.

(h) Write down the equation of the asymptote of the curve $xy - 3x - 4y = 0$ which is parallel to the x-axis.

(i) Write down the intrinsic equation of the catenary $y = c\cosh\left(\frac{x}{c}\right)$.

(j) Find the surface area of the solid generated by the curve $x^2 + y^2 = a^2$.

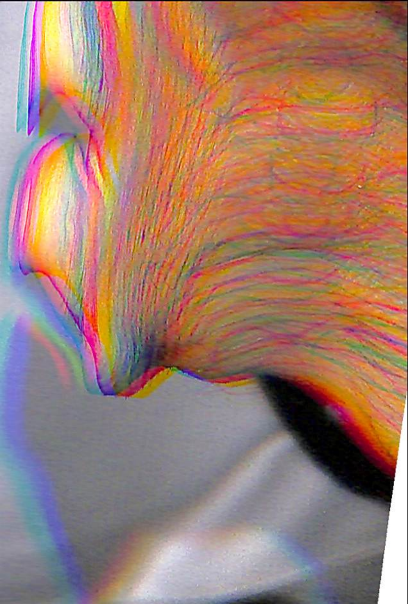
2. Answer the following questions : 2×5=10

(a) If $\cos(2x+b)$ find y_n

(b) If $y = f(x+ct) + g(x-ct)$, show that $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

(c) Prove that $\int_0^{\pi} x\cos^4 x \, dx = \frac{3\pi^2}{16}$

(d) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, prove that $I_n = \frac{1}{n-1} I_{n-2}$



(e) Show that the area of a loop of the curve $r = a\cos 2\theta$ is $\frac{\pi a^2}{8}$.

3. Answer the following questions : 5×4=20

(a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^2}$$

(b) Integrate $\int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$

(c) Trace the curve $y^2\left(a^2 + x^2\right) = x^2\left(a^2 - x^2\right)$

(d) Show that the area bounded by the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the co-ordinate axes is $\frac{1}{6}a^2$.

4. Answer **any two** questions : 5×2=10

(a) State and prove Leibniz's theorem.

(b) Prove that

$$\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

where $y = \left(\sin^{-1} x\right)^2$.

(c) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

5. Answer **any two** questions : 5×2=10

(a) State and prove Euler's theorem on homogeneous functions for two variables.

(b) Prove that the radius of curvature for the curve $y = f(x)$ at the point $p(x, y)$ is given by

$$\rho = \frac{\left(1 + y_1^2\right)^{\frac{3}{2}}}{y_2} \text{ where } y_1 = \frac{dy}{dx},\ y_2 = \frac{d^2y}{dx^2}$$

(c) Find the asymptotes of the curve

$$y^3 + x^2y + 2xy^2 - y + 1 = 0$$

6. Answer **any two** questions : 5×2=10

(a) If $I_n = \int tan^n x dx$, $(n > 1)$

prove that $I_n = \frac{tan^{n-1} x}{n-1} - I_{n-2}$

Hence obtain $\int tan^3 x dx$.

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 - cos\theta)$ about the initial line.

(c) Examine for double points on the curve $x^2y + x^3y + 5x^4 = y^2$

7. Answer **any two** questions : 5×2=10

(a) Find the total length of the astroid

$$x^{2/3} + y^{2/3} = a^{2/3}$$

(b) Find the area included between the curve $y^2(a - x) = x^3$ and its asymptote.

(c) Find the position and nature of the multiple points on the curve

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$
