2018

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write nth derivative of $\log(ax+b)$.

(b) If $z = f(y/x)$, what is the value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

(c) Write the formula for radius of curvature of a Cartesian curve.

(d) Two curves $y = f(x)$ and $y' = g(x)$ intersect at the point (x_1, y_1). Find the condition that they cut orthogonally.

(e) Define double point of a curve.

(f) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = 4$ about X-axis?

(g) Write subnormal to the curve $y^2 = 4ax$ at any point (x, y).

(h) Write the value of $\int_0^{\pi/2} \cos^7 x\, dx$.

(i) Write the value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$$

(j) Write the maximum number of asymptotes of algebraic curve of nth degree.

2. Solve the following questions : 2×5=10

(a) If $y = e^{ax} \sin bx$, show that

$$y_2 - 2ay_1 + (a^2 + b^2) y = 0$$

(b) If $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0, \ x \neq 0, \ y \neq 0$$

(c) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(d) Find the value of $\int_0^{\pi} x\cos^4 x\, dx$.

(e) Find the equation of tangent to the curve $y = be^{-x/a}$ at the point, where it crosses the axis of y.

3. Answer the following questions : 5×2=10

(a) If $y = \sin^{-1} x$, find $(y_n)_0$ where n is odd.

(b) Obtain a reduction formula for $\int \sec^n x \, dx$.

4. Answer either part (a) or part (b) : 10

(a) (i) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

(ii) Find the points of inflexion of the curve $y(a^2 + x^2) = x^3$. 5+5=10

(b) (i) If $u = f(x, y)$, where $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(ii) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x-axis, show that its equation is

$$y\cos\phi - x\sin\phi = a\cos 2\phi.$$

5+5=10

(Continued)

5. Answer the following questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

(b) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6. Answer part (a) or part (b) : 10

(a) (i) Obtaining nth derivative of x^{2n}, prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \cdots = \frac{\lfloor 2n}{(\lfloor n)^2}$$

(ii) Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$. 5+5=10

(b) (i) Find the asymptotes of the curve

$$x^3 + y^3 - 3axy = 0$$

(ii) Trace the curve $r = a(1 + \cos\theta)$. 5+5=10

(Turn Over)

7. Answer any *two* questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{5+3\cos x}$$

(b) If $I_n = \int (a^2+x^2)^{n/2}dx$, show that

$$I_n = \frac{x(a^2+x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$

(c) Evaluate :

$$\int \frac{dx}{(x^2-2x+1)\sqrt{x^2-2x+3}}$$

8. Answer the following questions : 5×2=10

(a) Integrate :

$$\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$$

Or

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$$

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

Or

Find the surface area of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line.

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