2018

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : $1 \times 7 = 7$

(a) Write down the moment of inertia of a circular disc of mass M and radius a about an axis through its centre and perpendicular to its plane.

(b) Define radius of gyration of the rigid body about a line.

(c) State the principal axes of a rigid body at a point O of the body.

(d) A rigid body rotates with angular velocity $\vec{\omega}$ about a fixed axis and I denotes the moment of inertia of the body about the axis. Write down the expression for the kinetic energy of the body.

(e) What do you mean by holonomic system?

(f) Define conservative system.

(g) State the theorem of the principle of conservation of energy of a rigid body.

2. Answer the following questions : 2×4=8

(a) A rigid body consists of 3 particles of masses 3 units, 5 units and 2 units located at the points $(-1, 0, 1)$, $(2, -1, 3)$ and $(-2, 2, 1)$ respectively. Find the moments of inertia about (i) the y-axis and (ii) the z-axis.

(b) A body with one point fixed rotates with angular velocity $(0, 0, 2)$. Find the magnitude of the velocity of a particle of mass m of the body located at the point $(3, -4, 1)$.

(c) Find the number of degrees of freedom for a rigid body which has one point fixed but can move in space about this point.

(d) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

3. Answer the following questions : 5×3=15

(a) Find the moment of inertia of a hollow sphere of radius a and mass M about a diameter.

Or

If the moments and products of inertia of a body about three perpendicular concurrent axes are known, find the moment of inertia of the body about the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

(b) State d'Alembert's principle and use it to obtain the equations of motion of any rigid body.

Or

Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

(c) The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$ and $2b$ respectively. Show that the inclination to AB of one of the principal axes at A is

$$\frac{1}{2}\tan^{-1}\frac{3ab}{2(a^2 - b^2)}$$

4. Define impressed forces and effective forces. A uniform rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semivertical angle α. Show that

$$\omega^2 = \frac{3g}{4a\cos\alpha}$$ 2+8=10

Or

(a) A plank of mass m and length $2a$ is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M, starting from the upper end walks down the plank so that it does not move, show that he will reach the other end in time

$$\sqrt{\frac{4Ma}{(m+M)g\sin\alpha}}$$ 6

Or

If the moments and products of inertia of a body about three perpendicular concurrent axes are known, find the moment of inertia of the body about the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

(b) State d'Alembert's principle and use it to obtain the equations of motion of any rigid body.

Or

Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

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(b) A rod revolving on a smooth horizontal plane about one end, which is fixed, breaks into two parts; what is the subsequent motion of the two parts? 4

5. (a) A pendulum is supported at O and P is the centre of oscillation. Show that if an additional weight is rigidly attached at P, the period of oscillation is unaltered. 5

(b) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. 5

6. Derive the equations of motion of a rigid body in two dimensions when the forces acting on the body are finite. 10

Or

Write down the equations of motion of a rigid body in two dimensions under impulsive forces. Two equal uniform rods, AB and AC, are freely jointed at A, and are placed on a smooth table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to itself. Show that the resulting velocities of the middle points of AB and AC are in the ratio $2 : 7$. 3+7=10

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