

2018

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7

- (a) State one fundamental difference between a spherical triangle and a plane triangle.
- (b) Define polar triangle and its primitive triangle.
- (c) Mention one property of pole of a great circle.
- (d) What is the reason of the oval shape of the sun at rising?
- (e) Explain briefly the dynamical significance of Kepler's second law of motion.
- (f) Define orbital period and synodic period of a planet.
- (g) What is the declination of the pole of the ecliptic?

2. Answer the following questions : $2 \times 4 = 8$

- (a) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.
- (b) Prove that section of a sphere by a plane is a circle.
- (c) Show that right ascension α and declination δ of the sun is always connected by the equation

$$\tan \delta = \tan \epsilon \sin \alpha$$

ϵ being obliquity of the ecliptic.

- (d) The apparent altitude of a star due to refraction is 30° . Calculate the true altitude, the coefficient of refraction being $58.2''$.

3. Answer any *three* questions of the following :

$5 \times 3 = 15$

- (a) A port is in latitude l (north) and longitude λ (west). Show that the longitudes of places on the equator distance δ from the port are

$$\lambda \pm \cos^{-1}(\cos \delta \sec l)$$

- (b) What do you mean by rising and setting of a star? Prove that the hour angle H of a star at the time of setting is given by

$$\cos H = -\tan \phi \tan \delta$$

(c) Prove that

$$\cos v = \frac{\cos E - e}{1 - e \cos E} \text{ and } \sin v = \frac{\sqrt{1 - e^2} \sin E}{1 - e \cos E}$$

where v is the true anomaly and E is the eccentric anomaly at any position of a planet in its orbit.

- (d) If λ is the moon's celestial latitude at the instant of opposition, m and p her hourly motions in longitude and latitude respectively, s the hourly motion of the sun in longitude and C the sum of semi-diameters of the moon and that of the earth's shadow, show that the duration of the lunar eclipse is the difference between the two roots of t , given by

$$C^2 = (\lambda - pt)^2 + (m - s)^2 t^2$$

- (e) Define geocentric parallax. Show that geocentric parallax of a heavenly body varies as the sine of its apparent zenith distance.

4. Derive cosine formula related to a spherical triangle. In an equilateral spherical triangle ABC , prove the following :

$$(i) \quad 2 \cos \frac{a}{2} \cdot \sin \frac{A}{2} = 1$$

$$(ii) \quad \sec A = 1 + \sec a$$

$6 + 4 = 10$

5. (a) Derive the formula for refraction

$$R = k \tan \zeta$$

ζ being the apparent zenith distance of a heavenly body. Mention one limitation of this formula.

5+1=6

- (b) If z_1 and z_2 are the zenith distances of a star at upper and lower culmination respectively which are on opposite sides of the zenith, prove that

$$\delta = 90^\circ - \frac{z_1 + z_2}{2} \text{ and } \phi = 90^\circ - \frac{z_2 - z_1}{2}$$

where δ is the declination of the star and ϕ is the latitude of the place of observer.

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6. Define solar ecliptic limits. Show that the minimum angular distance D_0 of the moon and the sun for occurrence of solar eclipse will be

$$D_0 = \beta \cos j$$

where $\tan j = \frac{\tan i}{1 - m}$ the other symbols carry

usual meanings.

2+8=10

Or

Discuss the effects of annual parallax on celestial longitude and latitude.

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