2018

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : 1×4=4

   (a) What is analytic function?

   (b) Define a complex variable.

   (c) State De Moivre's theorem.

   (d) Find out $(i)^{1/2}$.

2. *(a)* Verify whether the function $f(z) = 3z^2 + 2$ is an analytical function or not. 2

(b) Demonstrate a graphical representation of complex variable through Argand diagram. 2

3. Find the complex conjugate of the functions

$$(x+iy)\cdot(a+ib) \text{ and } \frac{x-iy}{a+ib}$$

where x, y, a and b are real. 4

Or

Obtain the residues of the function

$$f(z) = \frac{1}{z^2+a^2} \quad a>0$$

4. Give the Laurent series expansion for $f(z)$. Obtain the Laurent expansion for the function

$$f(z) = \frac{1}{z(z-1)} \text{ about } z_0 = 0 \quad 2+3=5$$

5. *(a)* *(i)* Define isolated singular point and non-isolated singularity. 2

(ii) Using residue theorem, evaluate

$$\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$$ 5

Or

State and prove Taylor's theorem. 2+5=7

(b) State and prove Cauchy-Riemann conditions for analytical functions. 2+4=6

Or

Show that $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2} dx = \frac{\pi}{2}$. 6

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions/Choose the correct option : 1×4=4

(a) What is reversed effective force?

(b) What do you mean by holonomic constraint?

(c) For a conservative system, the potential energy does not depend upon

(i) force

(ii) generalised velocity

(iii) generalised coordinate

(iv) None of the above

(d) If a coordinate does not appear in Lagrangian, then it is called

(i) cyclic

(ii) non-cyclic

(iii) free

(iv) holonomic

7. Answer any *two* of the following questions : 2×2=4

(a) Define virtual displacement and discuss its significance.

(b) State and explain Hamilton's principle.

(c) Show that in a central force field the angular momentum of a particle is conserved.

(d) Mention two properties of Poisson bracket with proof.

8. Answer any *two* of the following questions : 3×2=6

(a) Show that the motion of a particle under central force always takes place in a plane.

(b) Find an expression for centripetal acceleration for a bead sliding on a uniformly rotating wire.

(c) Show that Hamiltonian H is a constant of motion if the Lagrangian L is not an explicit function of time.

9. (a) Set up the Lagrangian for a simple pendulum and hence obtain equation describing its motion. 4

Or

Find the equation of motion of a system with the following Lagrangian :

$$L = \frac{1}{2} e^{\alpha t} (\dot{x}^2 - \omega^2 x^2)$$

where α and ω are constants.

(b) Establish the differential equation for the orbit of a particle under central force. 4

Or

Set up Lagrangian equation for an Atwood machine and find an expression for its acceleration.

10. What is d'Alembert's principle? Obtain Lagrange's equation of motion for a conservative system using d'Alembert's principle. 2+6=8

Or

Define Hamiltonian of a system and establish Hamilton's canonical equations. 2+6=8

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