

2019

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7

- (a) Write down the moment of inertia of a solid sphere of radius a and mass M about a diameter.
- (b) Define equimomental systems.
- (c) State the theorem of parallel axes on moment of inertia.
- (d) Define the centre of oscillation of a compound pendulum.
- (e) What is the principle of conservation of energy?
- (f) A particle moving freely in space requires three coordinates (x, y, z) , to specify its position. What is the degree of freedom of the particle?
- (g) What are generalized coordinates?

2. Answer the following questions : $2 \times 4 = 8$

(a) A particle of mass 4 units is placed at the point $(-1, -1, 1)$. What is the product of inertia of the particle about $OX - OY$; and $OY - OZ$?

(b) A particle of mass 3 units is located at the point $(2, 0, 0)$. The particle rotates about O with angular velocity $\vec{\omega} = \hat{k}$. Find the angular momentum of the particle about O .

(c) A rigid body with one point fixed rotates with angular velocity $\vec{\omega}$ and has angular momentum $\vec{\Omega}$. Prove that the kinetic energy is given by

$$T = \frac{1}{2} (\vec{\omega} \cdot \vec{\Omega})$$

(d) A particle of mass m moves in a conservative force field. Write the Lagrangian function.

3. Answer the following questions : $5 \times 3 = 15$

(a) Show that the moment of inertia of a rectangular lamina of mass M and sides $2a, 2b$ about a diagonal is

$$\frac{2M}{3} \frac{a^2 b^2}{a^2 + b^2}$$

Or

Find the product of inertia of a semi-circular wire about diameter and tangent at its extremity.

(b) State and prove d'Alembert's principle.

(c) Show that the momental ellipsoid at the centre of an elliptic plate is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \left(\frac{1}{a^2} + \frac{1}{b^2} \right) z^2 = \text{constant}$$

Or

Show that a uniform rod of mass M is kinetically equivalent to three particles, rigidly connected and situated one at each end of the rod and its middle point, the masses of the particles being $\frac{1}{6}M$,

$$\frac{1}{6}M \text{ and } \frac{2}{3}M.$$

4. A rod of length $2a$, is suspended by a string of length l , attached to one end, if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that

$$\frac{3P}{a} = \frac{(4 \tan \theta - 3 \tan \phi) \sin \phi}{(\tan \phi - \tan \theta) \sin \theta}$$

10

Or

(a) A rough uniform board of mass m and length $2a$, rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance through which the board moves in this time.

6

- (b) A circular board is placed on a smooth horizontal plane and a body runs round the edge of it at a uniform rate. What is the motion of the board? 4

5. (a) Prove that the time of complete oscillation of a compound pendulum is

$$2\pi \sqrt{\frac{k^2}{gh}}$$

where k is the radius of gyration of the body about a fixed axis and h is the distance of centre of inertia of the body from the fixed axis. 5

- (b) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion. 5

6. A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motion. 10

Or

Obtain the equation of motion of a rigid body under impulsive forces. 10

★ ★ ★

- (b) A circular board is placed on a smooth horizontal plane and a body runs round the edge of it at a uniform rate. What is the motion of the board?

4

5. (a) Prove that the time of complete oscillation of a compound pendulum is

$$2\pi \sqrt{\frac{k^2}{gh}}$$

where k is the radius of gyration of the body about a fixed axis and h is the distance of centre of inertia of the body from the fixed axis.

5

- (b) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion.

5

6. A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motion.

10

Or

Obtain the equation of motion of a rigid body under impulsive forces.

10

★ ★ ★