2019

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) If $x = 2 \cdot 536$, find the absolute and the relative errors if x is truncated to two-decimal digits.

(b) Define 'truncation' error.

(c) Write the following numbers correct up to four significant figures :

0·00408, 0·10254

(d) Evaluate $\Delta^2\left(\frac{1}{x-1}\right)$, taking $h = 1$.

(e) Show that $\Delta \cdot \nabla \equiv \Delta - \nabla$.

(f) State Lagrange's interpolation formula for $(n+1)$ unequally spaced arguments.

(g) Write Simpson's one-third rule in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in 1·8921 given its relative error as $0 \cdot 1 \times 10^{-2}$.

(b) Prove that

$$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

(c) Write the proper numerical differentiation formula to find the first derivative of a function $f(x)$ at a point x near the middle of a given set of tabulated values.

(d) Write a short note on numerical integration.

3. Answer the following questions : 5×3=15

(a) If $\Delta x = 0 \cdot 005$, $\Delta y = 0 \cdot 001$ be the absolute errors in $x = 2 \cdot 11$ and $y = 4 \cdot 15$, find the relative error in the computation of $x + y$.

(b) Use the method of separation of symbols to prove the following identity :

$$u_0 - u_1 + u_2 - \cdots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \cdots$$

Or

Find the missing entry in the following table :

x	0	1	2	3	4
y_x	1	3	9	—	81

(c) Find the cubic polynomial corresponding to the following data and hence evaluate $f(2 \cdot 4)$:

x	0	1	2	3	4
$f(x)$	1	2	1	10	41

Or

Prove that the nth order divided differences of a polynomial of degree n in x are constants.

4. Answer any *one* part :

(a) *(i)* Apply Gauss's forward formula to find the value of u_9, if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{12} = 35$, $u_{16} = 40$.

(ii) From the following data, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1$:

x	1	2	3	4	5	6
y	1	8	27	64	125	216

5+5=10

(b) *(i)* From the following table, find the value of y when $x = 1{\cdot}62$, using Stirling's formula :

x	1·3	1·4	1·5	1·6
$y = f(x)$	0·26236	0·33647	0·40547	0·47000

x	1·7	1·8	1·9
$y = f(x)$	0·53063	0·58779	0·64185

(ii) Calculate the value of $\int_0^1 \frac{x}{1+x}\,dx$ correct up to three significant figures taking six intervals, using the trapezoidal rule. 5+5=10

5. Answer any *one* part :

(a) *(i)* A rod is rotating in a plane about one of its ends. If the following table gives the angle θ (in radians) through which the rod has turned for different values of time t (in seconds), find its angular velocity at $t = 7$ sec :

t (in seconds)	0·0	0·2	0·4	0·6	0·8	1·0
θ (in radians)	0·0	0·12	0·48	1·10	2·0	3·20

(ii) Find the maximum and minimum values of y from the following data :

x	0	1	2	5
y	2	3	12	147

5+5=10

(b) (i) The velocity v of a particle moving in a straight line covering a distance x in time t are given by the following table :

x	0	10	20	30	40
v	45	60	65	54	42

Find the time taken to traverse the distance of 40 units.

(ii) Find the value of $\int_0^{0.6} e^x dx$ by Simpson's $\frac{1}{3}$rd rule, dividing the range into six equal parts. 5+5=10

6. Answer any *one* part :

(a) (i) Give the geometrical interpretation of Newton-Raphson method.

(ii) Find the root of $\tan x + x = 0$, using bisection method, lying between 2 and 2·1. (Perform five iterations) 5+5=10

(b) (i) Find a negative root of the equation $x^3 - \sin x + 1 = 0$ correct to three decimal places, using Newton-Raphson method.

(ii) Derive the rate of convergence of Newton-Raphson method. 5+5=10

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