Department
6.1, 6.2, 6.3, 6.4.

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MATHEMATICS

(Major)

Paper : 6.1

(Hydrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Define surface of equal pressure.

(b) State Boyle's law.

(c) Define absolute zero of temperature.

(d) Define centre of pressure of a plane area immersed in a fluid.

(e) What is internal energy?

(f) Define the term convective equilibrium for a gas.

(g) Define surface of floatation.

2. Answer the following questions : 2×4=8

(a) Obtain the differential equations of the lines of force at any point (x, y, z).

(b) Prove that the pressure at any point varies as the depth of the point from the surface when there is no atmospheric pressure.

(c) If ρ_0 and ρ be the densities of a gas at 0° and $t°$ centigrade respectively, then establish the relation $\rho_0 = \rho(1 + \alpha t)$ where

$$\alpha = \frac{1}{273}$$

(d) Show that the positions of equilibrium of a body floating in a homogeneous liquid are determined by drawing normals from G, the centre of mass of the body, to the surface of buoyancy.

3. Answer any *three* parts : 5×3=15

(a) If a mass of fluid is at rest under the action of given forces, obtain the equation which determines the pressure at any point of the fluid.

(b) A circular area of radius a is immersed with its plane vertical and centre at a depth h. Find the depth of the centre of pressure.

(c) If the absolute temperature T at a height z is a function $f(z)$ of the height, show that the ratio of the pressures at two heights z_1 and z_2 is given by

$$\log \frac{P_2}{P_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{f(z)}$$

R being the constant in the equation $p = R\rho T$, where P_1, P_2 are the pressures at heights z_1 and z_2 respectively.

(d) Prove that a floating body is in stable or unstable equilibrium as the metacentre is above or below the centre of gravity of the body.

(e) Masses m, m' of two gases in which the ratios of the pressure to the density are respectively k and k' are mixed at the same temperature. Prove that the ratio of the pressure to the density in the compound is

$$\frac{mk + m'k'}{m + m'}$$

4. Answer *either* (a) *or* (b) :

(a) (i) If a fluid is at rest under the action of the forces X, Y, Z per unit mass, find the differential equations of the curves of equal pressure and density. 5

(ii) If the components parallel to the axes of the forces acting on the element of fluid at (x, y, z) be proportional to $y^2 + 2\lambda yz + z^2$, $z^2 + 2\mu zx + x^2$, $x^2 + 2\nu xy + y^2$, show that, if equilibrium is possible, we must have $2\lambda = 2\mu = 2\nu = 1$. 5

(b) (i) A mass of fluid rests upon a plane subject to a central attractive force $\frac{\mu}{r^2}$, situated at a distance c from the plane on the side opposite to that on which is the fluid. Show that the pressure on the plane is

$$\frac{\pi \rho \mu (a-c)^2}{a}$$

a being the radius of the sphere of which the fluid, on the plane in the form of a cap is a part. 5

(Continued)

(ii) If $X = y(y+z)$, $Y = z(z+x)$ and $Z = y(y-x)$, prove that surfaces of equal pressure are the hyperbolic paraboloids $y(x+z) = c(y+z)$ and the curves of equal pressure and density are given by $y(x+z) =$ constant and $y + z =$ constant. 5

5. Answer *either* (a) *or* (b) :

(a) (i) If an area is bounded by two concentric semi-circles with their common bounding diameter in the free surface, prove that the depth of the centre of pressure is

$$\frac{3\pi(a+b)(a^2+b^2)}{16(a^2+ab+b^2)}$$

where a and b are the radii of the outer and inner circles respectively. 5

(ii) A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure. 5

(Turn Over)

(b) (i) A hollow hemispherical shell has a heavy particle fixed to its rim, and floats in water with the particle just above the surface and with the plane of the rim inclined at an angle 45° to the surface. Show that the weight of the hemisphere is to the weight of the water which it would contain $4\sqrt{2}-5:6\sqrt{2}$. 5

(ii) A right cone is totally immersed in water, the depth of the centre of its base being given. Prove that p, p', p'' being the resultant pressures in its convex surface, when the sines of the inclination of its axis to the horizontal are s, s', s'' respectively

$$p^2(s'-s'')+p'^2(s''-s)+p''^2(s-s')=0$$ 5

6. Answer *either* (a) or (b) :

(a) (i) Show that for a perfect gas C_P is greater than C_V, where C_P is the specific heat keeping pressure as constant and C_V is the specific heat keeping volume as constant. 5

(ii) A gas satisfying Boyle's law $p=k\rho$ is acted on by the forces

$$X=-\frac{y}{x^2+y^2},\quad Y=\frac{x}{x^2+y^2}$$

Show that the density varies as $e^{\theta/k}$, where $\tan\theta=\frac{y}{x}$. 5

(b) (i) Prove that a floating body is in stable or unstable equilibrium as the metacentre is above or below the centre of gravity of the body. 5

(ii) A cone of given weight and volume floats with its vertex downwards, prove that the surface of the cone in contact with the liquid is least when its vertical angle is

$$2\tan^{-1}\frac{1}{\sqrt{2}}$$ 5

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