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3 (Sem-6/CBCS) PHY HE 5

2024

PHYSICS

(Honours Elective)

Paper : PHY-HE-6056

(Classical Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 10 = 10$

(a) Write the necessary and sufficient condition for force F to be conservative.

(b) A system of 4 particles has 10 equations of constraints and requires 2 generalized coordinates. Are the constraints holonomic or non-holonomic?

(c) Write the expression for Hamiltonian of a free particle in spherical polar coordinates.

Contd.

- (d) State one advantage of Lagrangian formulation over Newtonian formulation.
- (e) What is called gyro frequency? Write down its expression.
- (f) Write down the relativistic form of Newton's second law of motion.
- (g) What is the significance of Reynold's number?
- (h) Write down the Newton's law of viscous flow in streamline motion and hence define the coefficient of viscosity.
- (i) Express equation of continuity in terms of four current density vector.
- (j) Write down the Lorentz transformation equations of energy and momentum.

2. Answer the following questions: $2 \times 5 = 10$

- (a) "Magnetic field changes the velocity of a charged particle without changing its speed." Explain the statement.
- (b) Show that Lagrangian and Newtonian equations of motion are equivalent.
- (c) What are the different types of relativistic optical Doppler effects?

(d) State and explain postulates of special theory of relativity.

(e) A tube of radius r and length l is connected in series with another of radius $\frac{r}{2}$ and length $\frac{l}{4}$. If the pressure across the two tubes taken together is p , deduce the pressures across the tubes separately.

3. Answer **any four** from the following questions: $5 \times 4 = 20$

(a) Show that the path of a charged particle in a uniform magnetic field, in general, is a helix. Under what condition is this path reduced to a circle? $4 + 1 = 5$

(b) Derive Lagrange's equations of motion for a conservative system using D' Alembert's principle.

(c) What do you mean by the element of proper time?

Using four vector expressions show that $E^2 = p^2 c^2 + m_0^2 c^4$, where symbols have their usual meanings. $1 + 4 = 5$

(d) Write brief notes on space-like and time-like intervals.

(e) Express Lorentz transformations of space and time in four vector form.

(f) (i) Using Euler-Lagrange equation prove that 'the shortest distance between two points in a plane is a straight line'. 4

(ii) State Hamilton's principle. 1

4. Answer the following questions: $10 \times 4 = 40$

(a) (i) Show that Hamiltonian H is a constant of motion if the Lagrangian L is not an explicit function of time. 2

(ii) Derive Hamilton's canonical equations and use these to obtain the equation of motion of a simple pendulum. $4 + 4 = 8$

Or

(b) (i) What do you mean by stable and unstable equilibria? $2 + 2 = 4$

(ii) Obtain Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium. 4

(iii) Write the principle of virtual work in terms of independent generalized coordinates. 2

(c) (i) The equation of the orbit of a particle under the action of a central force is $r = 2a \cos \theta$.

Show that the force F is inversely proportional to r^5 . 4

(ii) Prove the conservation of energy of a particle directly from its equation of motion in a central force field. 4

(iii) The nature of orbit is determined by the value of its eccentricity

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{\mu k^2}}$$

where symbols have their usual meanings.

Mention the various special cases depending upon the values of E and ε . 2

Or

(d) (i) A proton with initial velocity of $5 \times 10^6 \text{ ms}^{-1}$ passes through an electric field (transverse) of 200 volt/cm . Calculate the transverse deflection in travelling a distance of 1 m . 3

(ii) Obtain equations of motion of a system of coupled simple pendulums by setting Lagrangian of the entire system. 7

(e) (i) Explain the concept of twin paradox with the help of space-time diagram. 5

(ii) Draw a neat diagram of light cones indicating past and future. Show the world lines in it.

Can the tangent to the world line of a massive particle at a point has an angle equal to or more than 45° ? Explain. $3+2=5$

Or

(f) (i) What are called cyclic or ignorable coordinates? If a system undergoes translatory motion along a cyclic generalized coordinate q_k , will the Lagrangian of the system be affected? 2

(ii) Obtain the Lagrange's equation of motion for an electrical circuit comprising an inductance L and capacitance C . The capacitor is charged to q coulombs and the current flowing in the circuit is i amperes. 3

(iii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four-dimensional Minkowski space. 5

(g) (i) State length contraction and time dilation. How are the phenomenon of length contraction and time dilation interpreted on space-time diagram? $2+(3+3)=8$

(ii) Calculate the velocity which 1 amu of mass will have, if it had a kinetic energy 3 times the rest mass energy. 2

Or

(h) (i) The rate of a liquid through a capillary tube is $V = \frac{P\pi r^4}{8\eta l}$ with usual notations. Deduce the relation stating clearly the conditions under which it holds. 6

(ii) Discuss the corrections to be applied to Poiseuille's equation. 4

(iii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four-dimensional Minkowski space. 5

(iv) State length contraction and time dilation. How are the phenomenon of length contraction and time dilation interpreted on space-time diagram? $2 + (3+3) = 8$

(v) Calculate the velocity which 1 gram of mass will have, if it had a kinetic energy 3 times the rest mass energy. 3+2+3